

# Sparsity regularization and graph-based representation in medical imaging

Ph.D. Thesis Defense

Katerina Gkirtzou

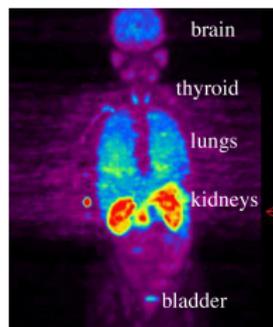
Supervisors : Nikos Paragios and Matthew Blaschko



17th December, 2013

# Medical Imaging

## PET

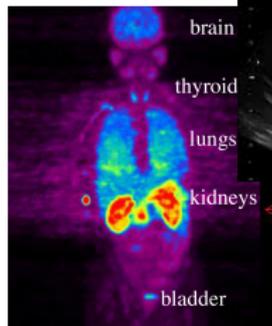


# Medical Imaging

## Ultrasound



## PET

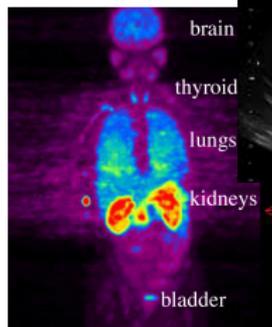


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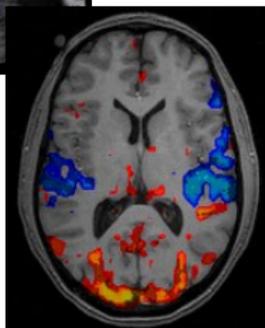
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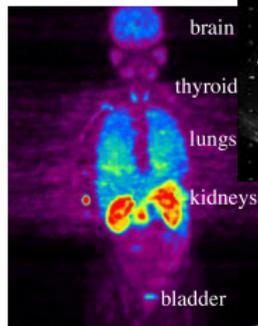


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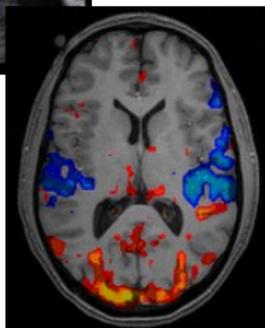
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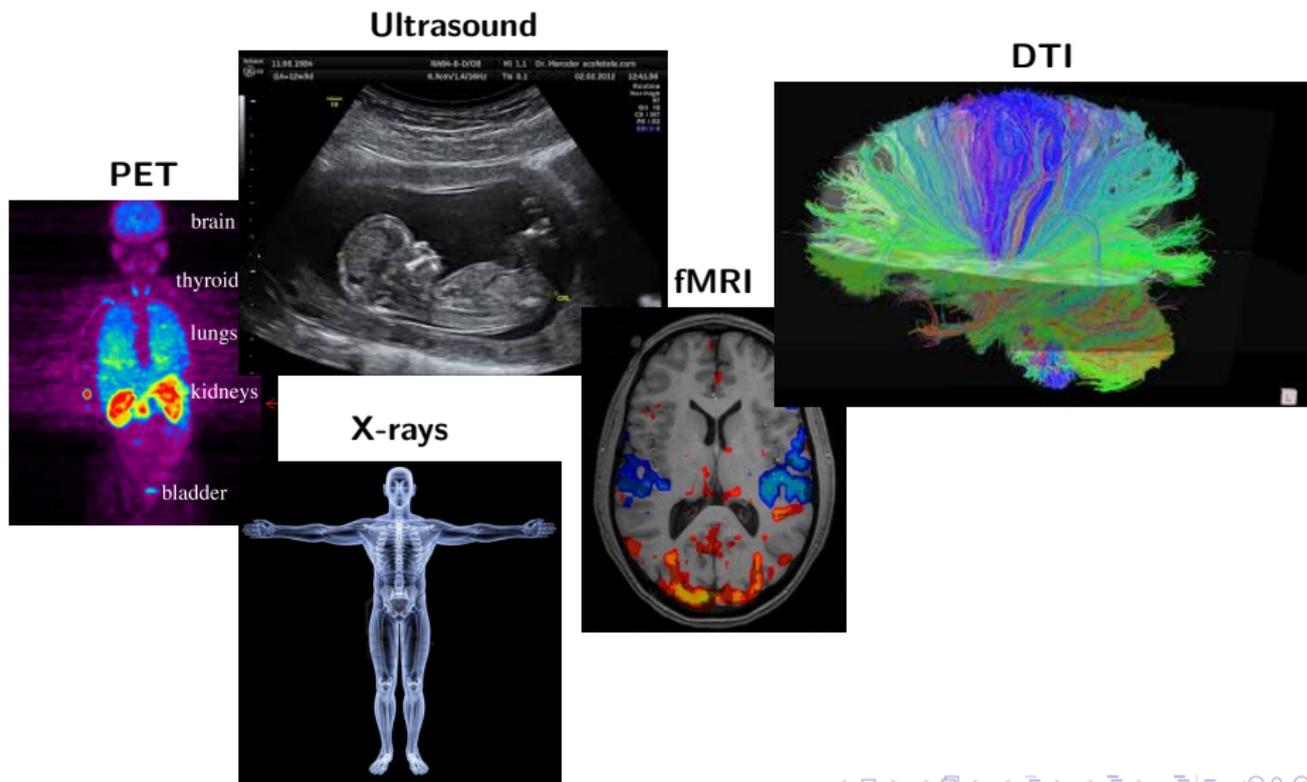
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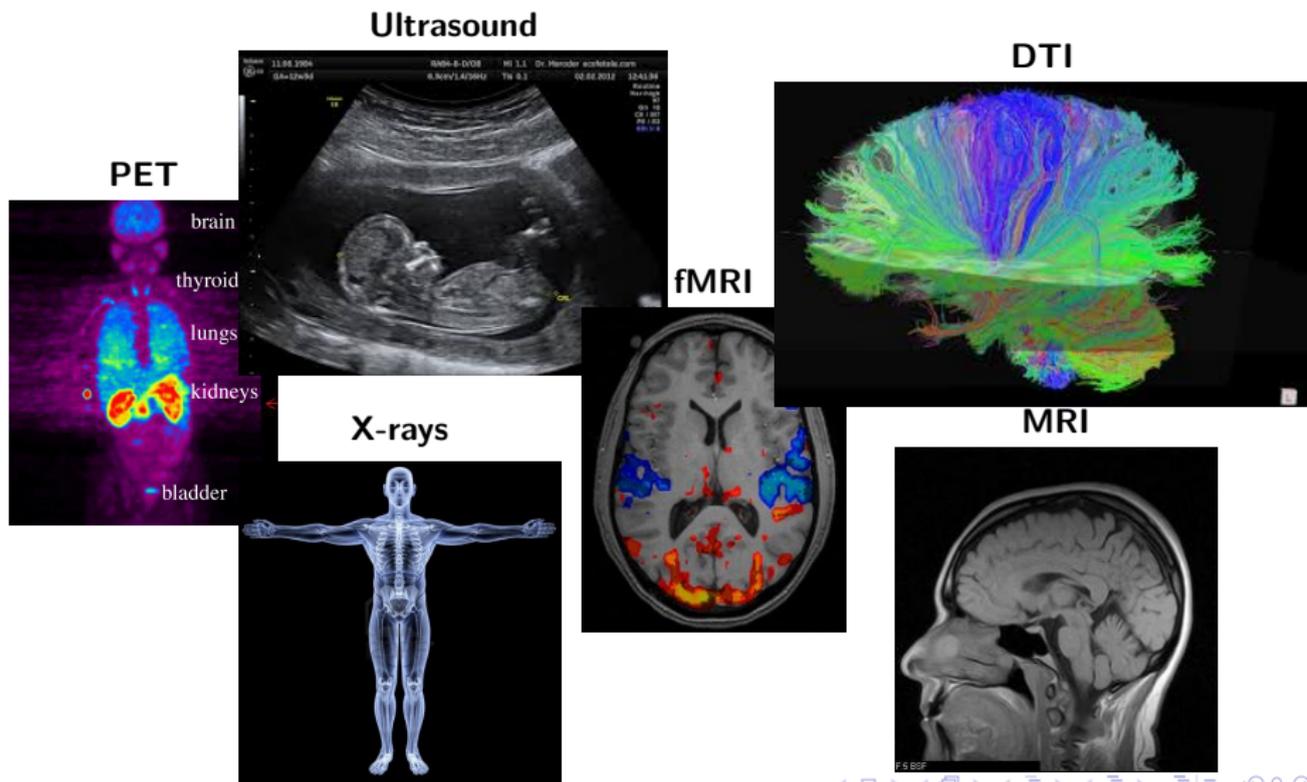
## X-rays



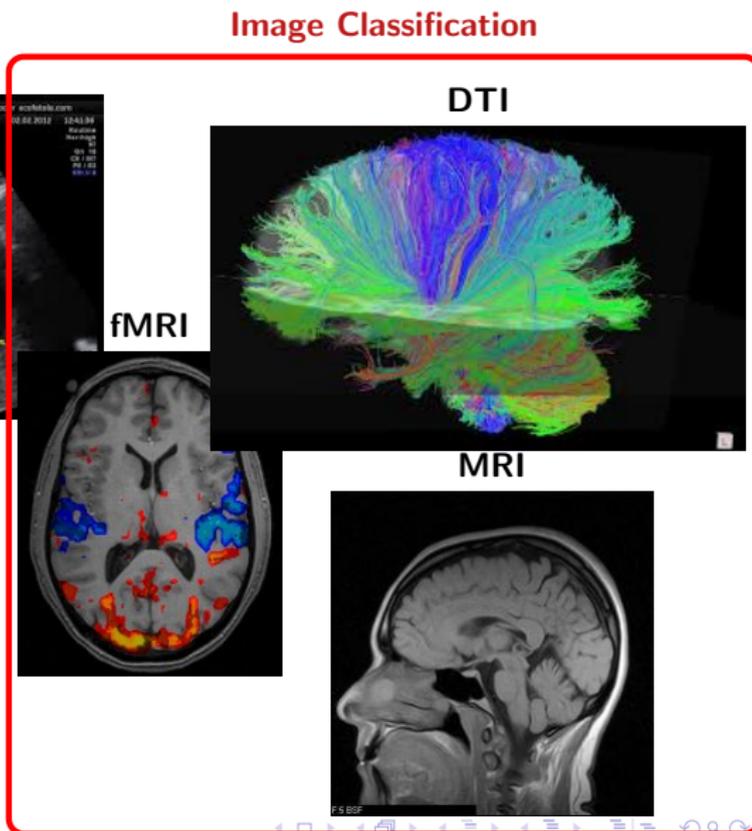
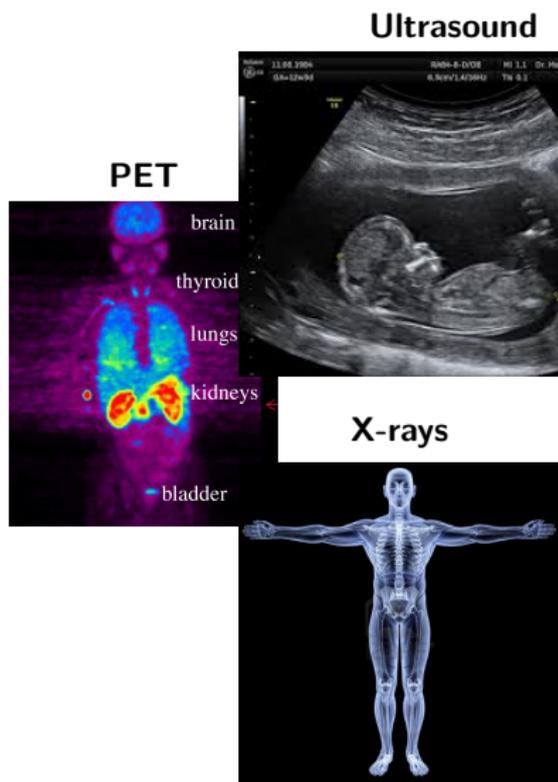
# Medical Imaging



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# Medical image classification

Given a set of  $n$  paired observations  $\{(\mathbb{I}_i, y_i)\}_{1 \leq i \leq n}$  where

- $\mathbb{I}_i$  is an medical image and
- $y_i \in \mathbb{R}$  is the classification label

the goal is to learn a *classification function*  $f$ .

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- 1 The representation of  $\phi(\mathbb{I})$ .
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- 2 Supervised statistical learning framework

$$\arg \min_{f \in \mathcal{F}} \lambda \Omega(f) + \overbrace{\frac{1}{n} \sum_{i=1}^n \mathbb{L}(f(\phi(\mathbb{I}_i)), y_i)}^{\text{Empirical Risk}}$$

where  $\mathbb{L}$  is the *loss function* and  $\lambda \Omega(f)$  is the regularization term.

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- 1 Introduction to Graphs
- 2 The pyramid quantized Weisfeiler-Lehman graph representation
  - Overview
  - The Weisfeiler-Lehman algorithm
  - The pyramid quantization strategy
  - A sequence of discretely labeled graphs
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- 3 Experiments
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# What is a graph and why is it interesting?

## Definition

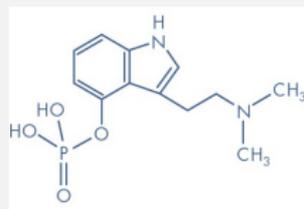
A *labeled graph*  $G$  is defined as a triplet  $(V, E, \mathcal{L})$ , where  $V$  is the *vertex set* and  $E \subseteq V \times V$  is the *edge set* which represents a binary relation on  $V$  and  $\mathcal{L} : X \mapsto \Sigma$  is a function assigning a label from an alphabet  $\Sigma$  to each element of the set  $X$ , which can be either  $V$ ,  $E$  or  $V \cup E$ .

# What is a graph and why is it interesting?

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## Areas of application



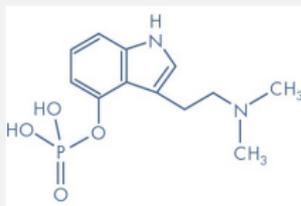
## Chemoinformatics

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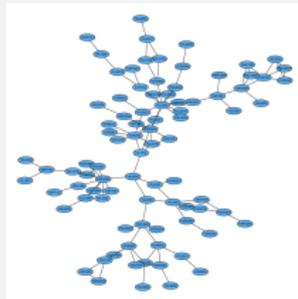
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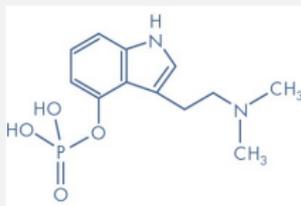
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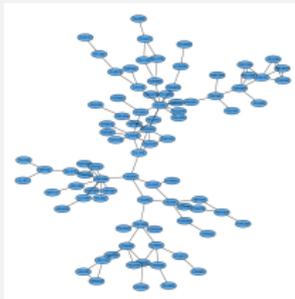
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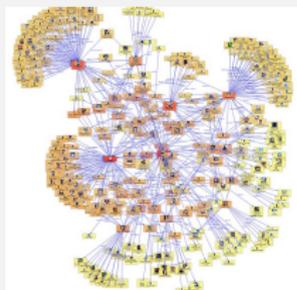
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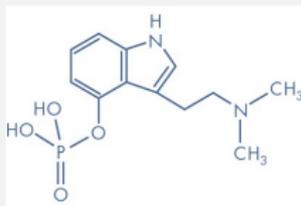
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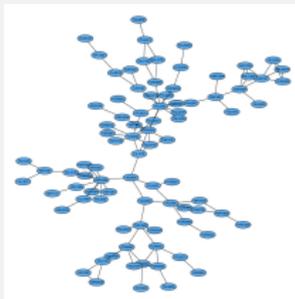
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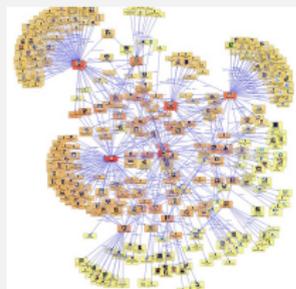
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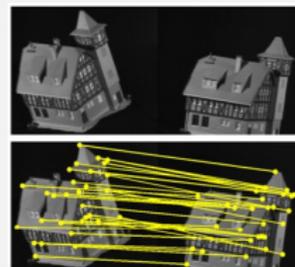
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**Computer Vision**

# Graph Comparison Problem

## Definition

Given a set  $\mathcal{G}$  of graphs, the problem of graph comparison is defined as a function

$$k : \mathcal{G} \times \mathcal{G} \mapsto \mathbb{R}$$

such that  $k(G, G')$  for  $G, G' \in \mathcal{G}$  quantifies the similarity of  $G$  and  $G'$ .

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## 1st Approach

- Graph Isomorphism
- Subgraph Isomorphism
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## 1st Approach

- Graph Isomorphism - No efficient algorithm is known
- Subgraph Isomorphism - Proven to be NP-complete
- Largest common subgraph - Proven to be NP-hard

# Graph Comparison Problem

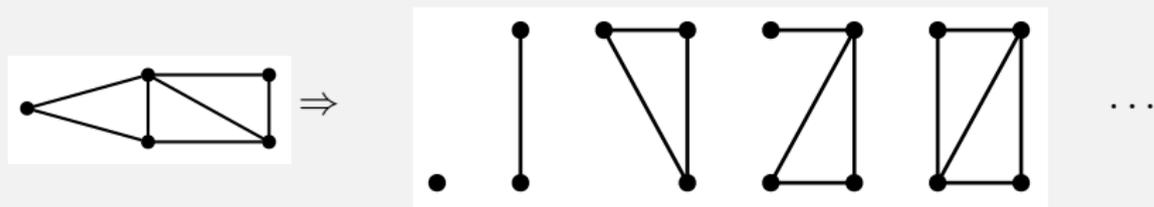
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## 2nd Approach - R-convolution Kernels



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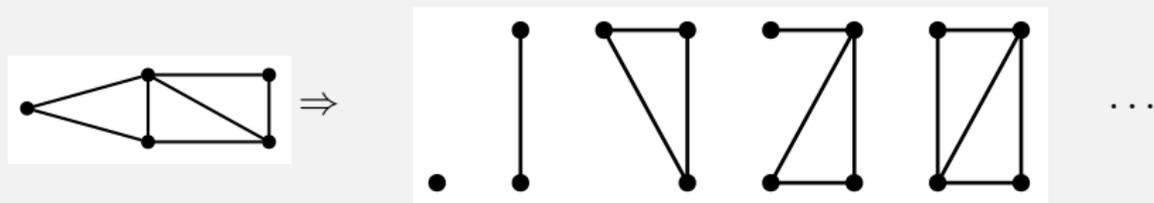
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## 2nd Approach - R-convolution Kernels



Calculating all subgraphs is at **least as hard** as deciding whether two graphs are isomorphic [Gärtner 03]

## Graph kernels

	Algorithm	Complexity <sup>1</sup>	Unlabeled	Discrete	Continuous	Vector
Subtree Patterns	[Gärtner 03]	$\mathcal{O}(n^2 v^6)$	✓	✓	✓	✓
	[Mahé 04]		✓	✓		
	[Vishwanathan 10]	$\mathcal{O}(n^2 v^3)$	✓	✓	✓	✓
Graphlets	[Borgwardt 05]	$\mathcal{O}(n^2 v^4)$	✓	✓	✓	✓
	[Ralaivola 05]		✓	✓		
	[Horváth 04]		✓	✓		
Paths	[Shervashidze 09]	$\mathcal{O}(vd^{k-1})$	✓			
	[Costa 10]		✓	✓		
	[Ramon 03]	$\mathcal{O}(n^2 v^2 h 4^d)$	✓	✓		
	[Bach 08]		✓	✓		
Patterns	[Mahé 09]		✓	✓		
	[Shervashidze 11]	$\mathcal{O}(nhe + n^2 hv)$	✓	✓		

<sup>1</sup>where  $n$  is the number of graphs,  $v$  is the maximal number of nodes,  $e$  is the maximal number of edges,  $h$  is the height of subtree patterns,  $d$  is the maximum degree and  $k$  is the size of graphlets.

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# Overview of the WLpyramid

Given a set  $\mathcal{G} = \{G_i = (V_i, E_i, \mathcal{L}_i)\}_{1 \leq i \leq n}$  where  $\mathcal{L}_i : V_i \mapsto \mathbb{R}^d$

- 1 A pyramid quantization of the label space.
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- 3 Produce subtree features with Weisfeiler-Lehman algorithm.
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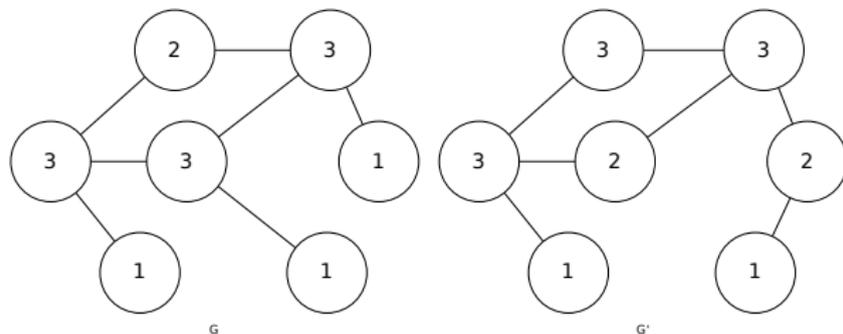
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## Why Weisfeiler-Lehman?

- 1 Computational time  $\mathcal{O}(nhe)$ 
  - $n$  the number of graphs
  - $e$  the maximal number of edges and and
  - $h$  the height subtree features.
- 2 Competitive accuracy in several classification benchmark data sets [Shervashidze 11].

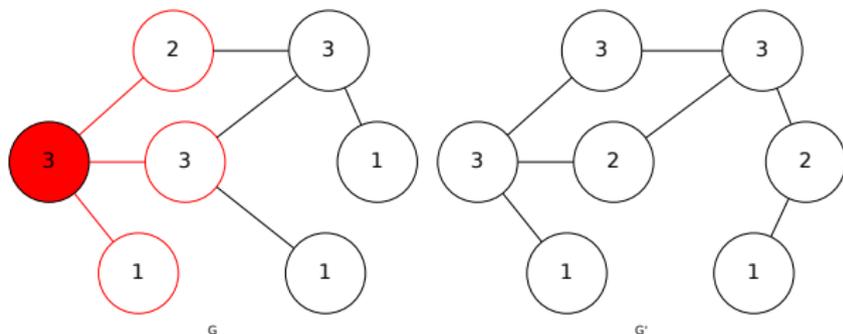
# The Weisfeiler-Lehman test of isomorphism [Weisfeiler 68]

Given labeled graphs  $G$  and  $G'$



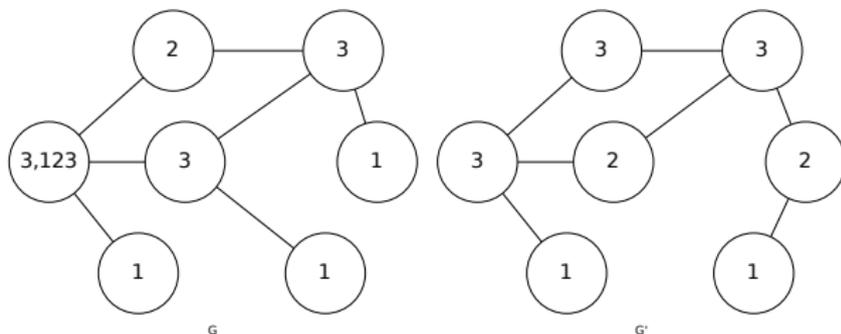
# The Weisfeiler-Lehman test of isomorphism [Weisfeiler 68]

## Multi-label determination of $G$ and $G'$



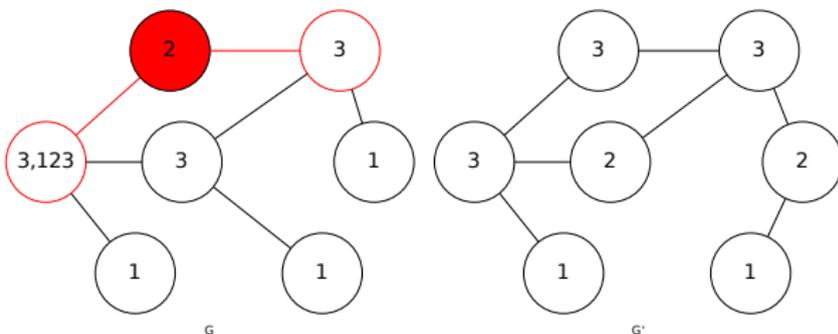
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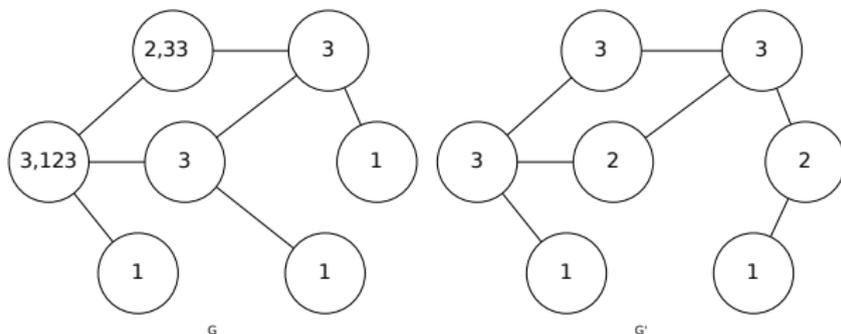
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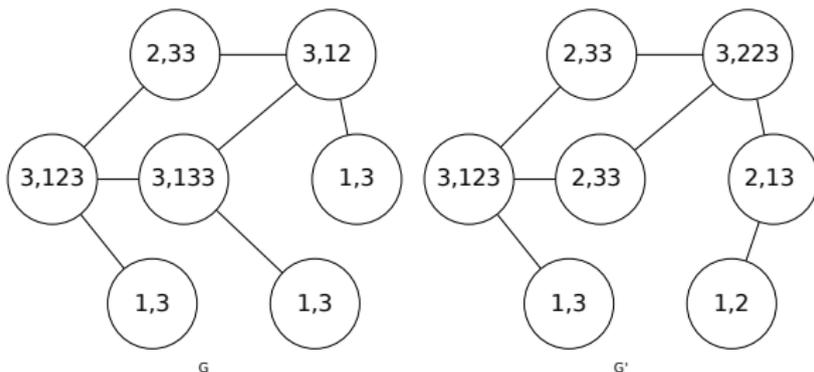
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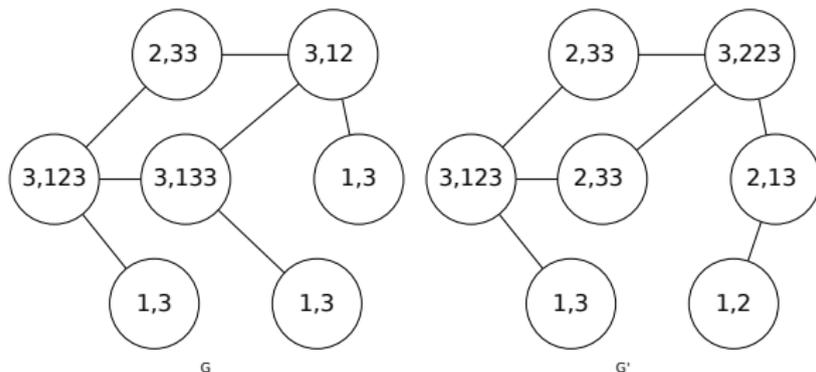
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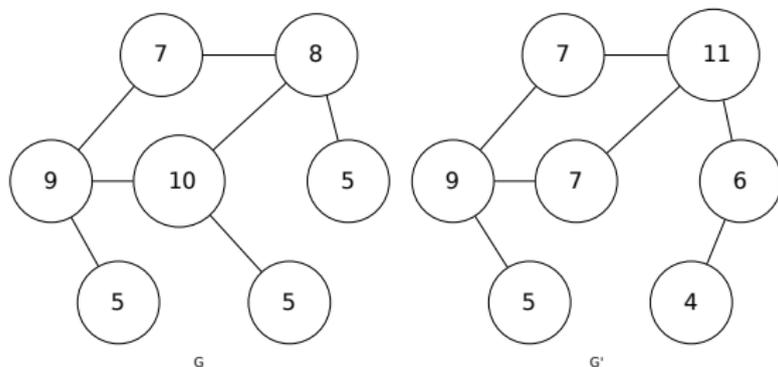


## Label compression via hashing

1,2	→	4	3,12	→	8
1,3	→	5	3,123	→	9
2,13	→	6	3,133	→	10
2,33	→	7	3,223	→	11

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## Relabeling graphs $G$ and $G'$

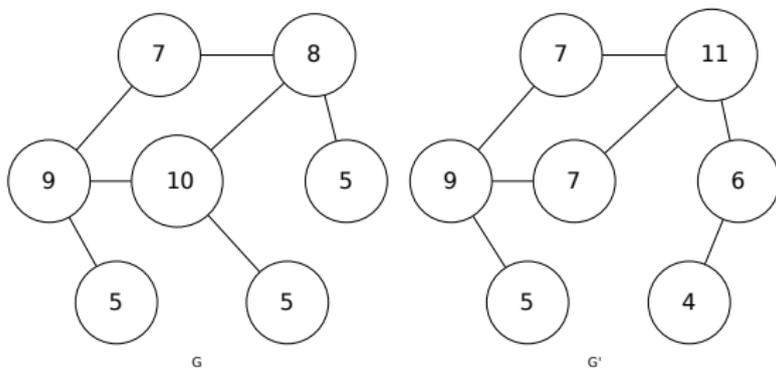


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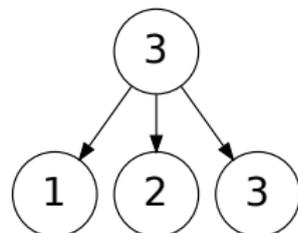
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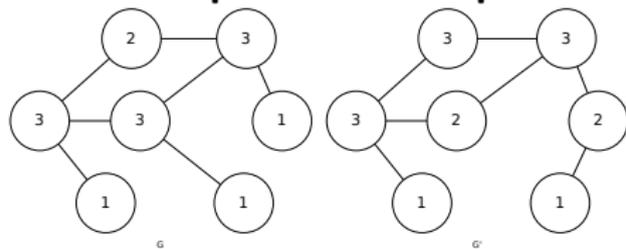
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## Subtree Pattern of Compressed label 9



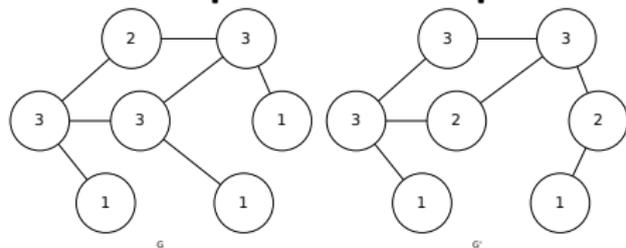
# Weisfeiler-Lehman subtree features

## Subtree patterns of depth 0.

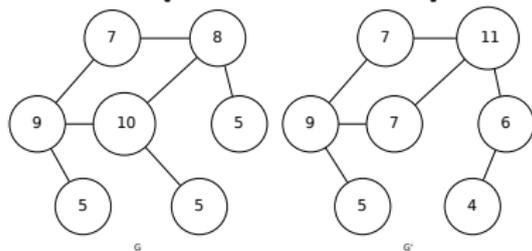


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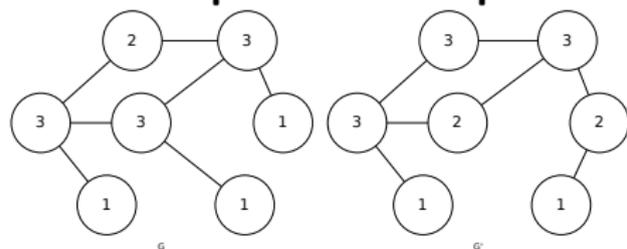


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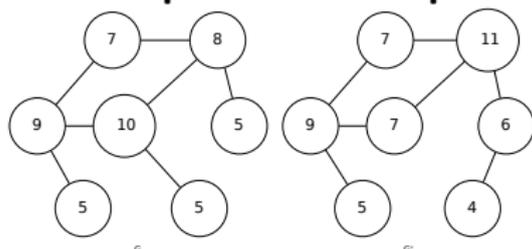


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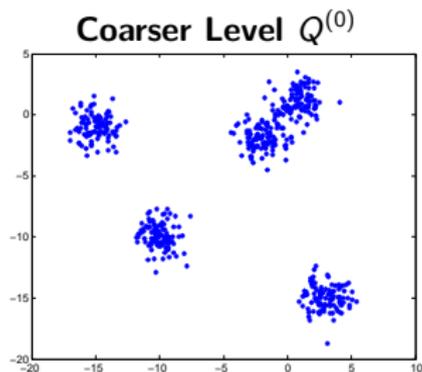


	Original node labels $\Sigma_0$	Compressed node labels $\Sigma_1$
Labels $\{\Sigma_0, \Sigma_1\}$	$\{1, 2, 3,$	$4, 5, 6, 7, 8, 9, 10, 11\}$
$\phi_{(1)}(G)$	$= (3, 1, 3,$	$0, 3, 0, 1, 1, 1, 1, 0)$
$\phi_{(1)}(G')$	$= (2, 2, 3,$	$1, 1, 1, 2, 0, 1, 0, 1)$

$\phi_{(h)}(G)$  are histograms of occurrences of the subtree patterns up to depth  $h$  in graph  $G$ .

# The pyramid quantization strategy

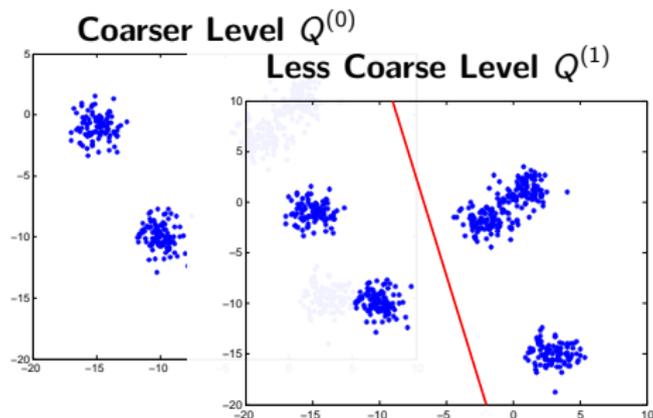
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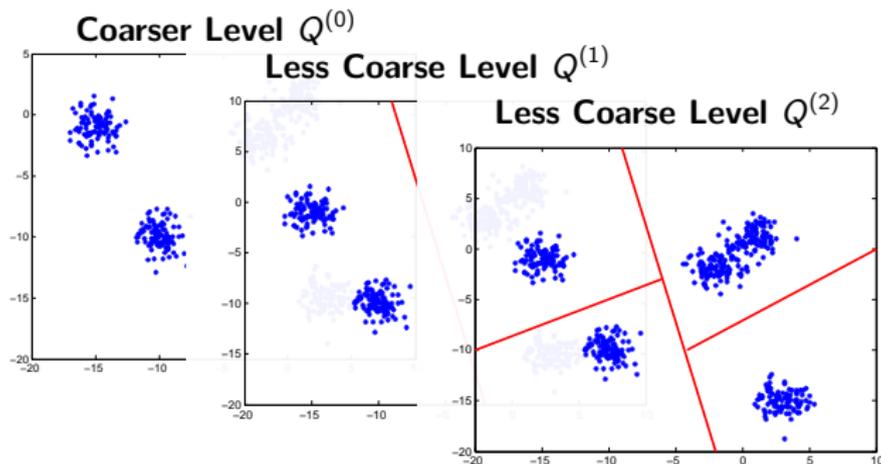
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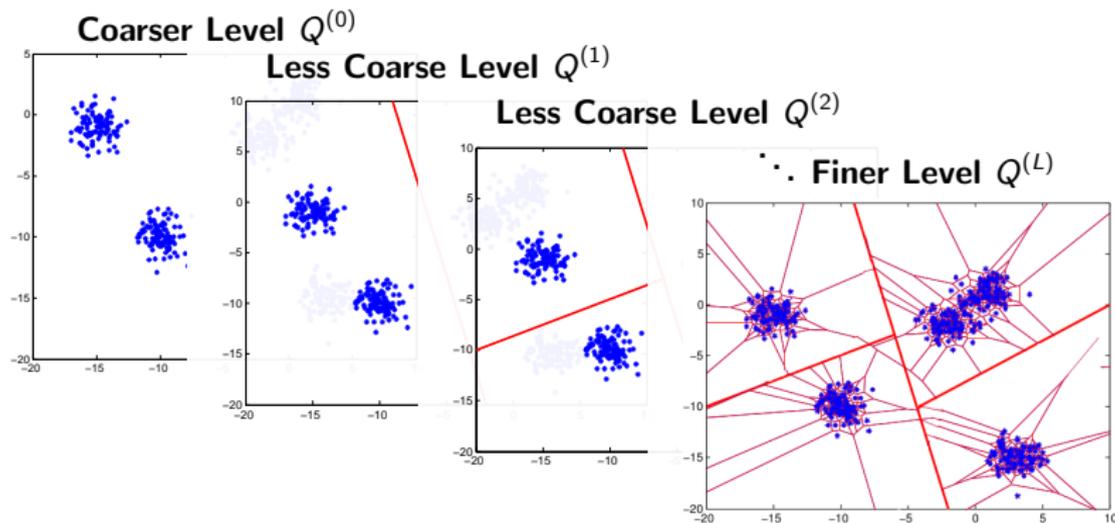
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# The pyramid quantization strategy

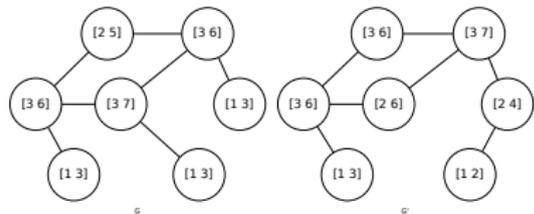
Given a set  $\mathcal{G} = \{G_i = (V_i, E_i, \mathcal{L}_i)\}_{1 \leq i \leq n}$  where  $\mathcal{L}_i : V_i \mapsto \mathbb{R}^d$



where  $L = \lceil \log_2 |V| \rceil$  and  $|V| = \sum_i^n |V_i|$  [Grauman 07].

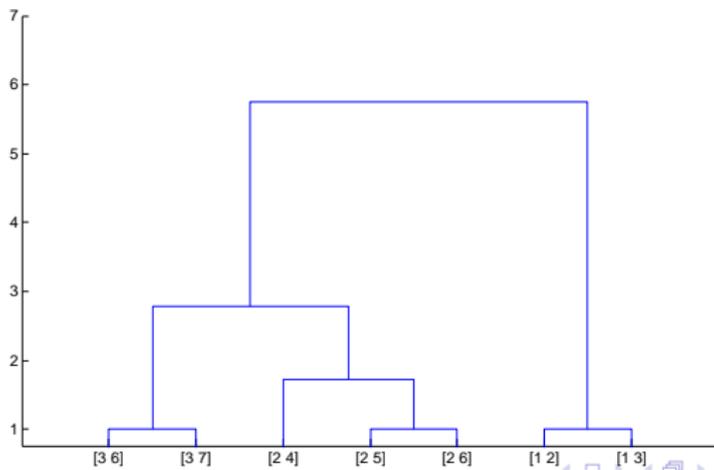
# Data guided pyramid quantization scheme

Given labeled graphs  $G$  and  $G'$



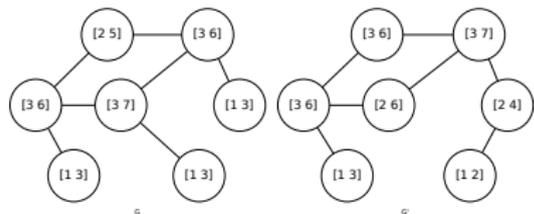
## Notes

- Ward's minimum variance method over the image of  $V$  under  $\mathcal{L}$ .



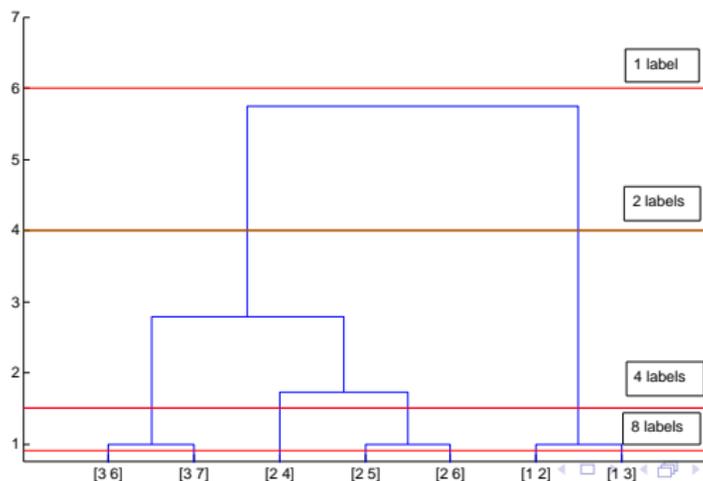
# Data guided pyramid quantization scheme

Given labeled graphs  $G$  and  $G'$



## Notes

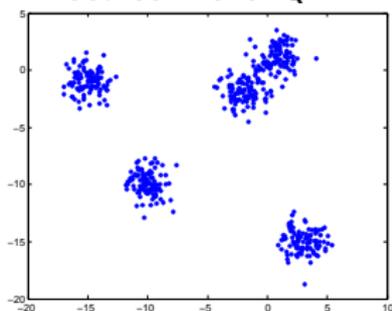
- Ward's minimum variance method over the image of  $V$  under  $\mathcal{L}$ .
- Selecting  $L = \lceil \log_2 D \rceil$ , where  $D \leq |V|$  the number of unique values in the image of  $V$  under  $\mathcal{L}$ .
- Each level  $l$  has  $2^l$  discrete labels.



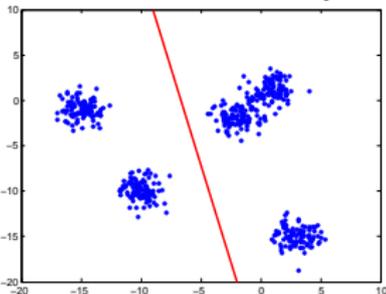
# Transform the initial graphs as a sequence of graphs

## The pyramid quantization of label space

Coarser Level  $Q^{(0)}$

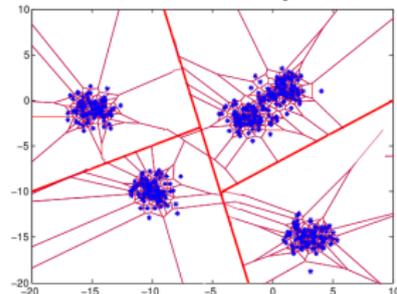


Less Coarse Level  $Q^{(1)}$



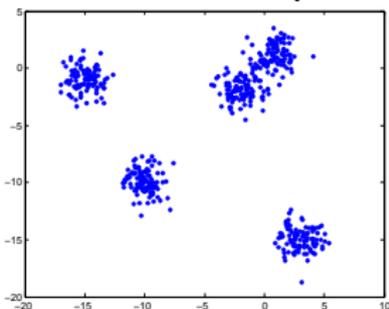
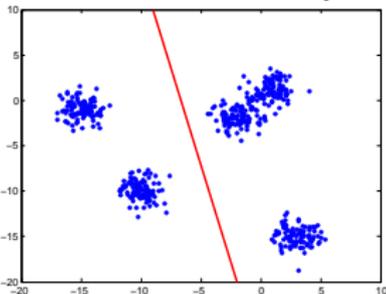
...

Finer Level  $Q^{(L)}$

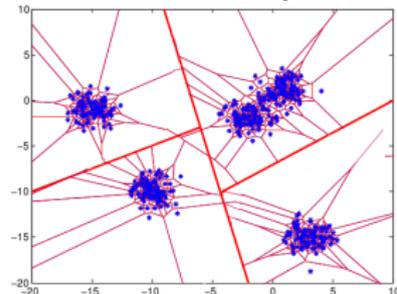


# Transform the initial graphs as a sequence of graphs

## The pyramid quantization of label space

Coarser Level  $Q^{(0)}$ Less Coarse Level  $Q^{(1)}$ 

...

Finer Level  $Q^{(L)}$ 

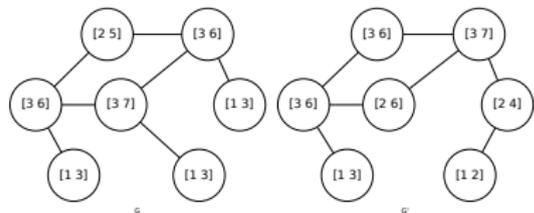
## Sequence of discretely labeled graphs

$$G = (V, E, \mathcal{L}) \xrightarrow[\forall l]{Q^{(l)} \circ \mathcal{L}} (G^{(0)}, \dots, G^{(L)}) \xrightarrow{\text{Increasing granularity}} \left( (V, E, \mathcal{L}^{(0)}), \dots, (V, E, \mathcal{L}^{(L)}) \right) \xrightarrow{\text{Increasing granularity}}$$

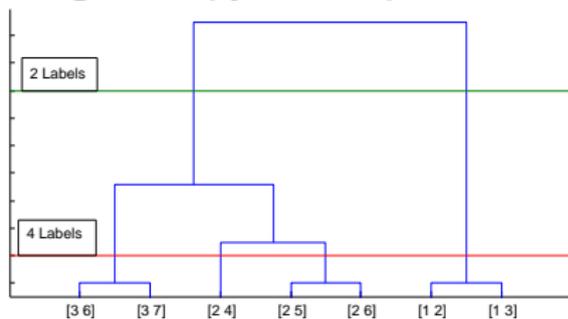
where  $\mathcal{L}^{(l)} : V \rightarrow \Sigma_0^{(l)}$ ,  $|\Sigma_0^{(l)}| = 2^l$  and  $l \in \{0, \dots, L\}$ .

# A sequence of discretely labeled graphs

Given labeled graphs  $G$  and  $G'$

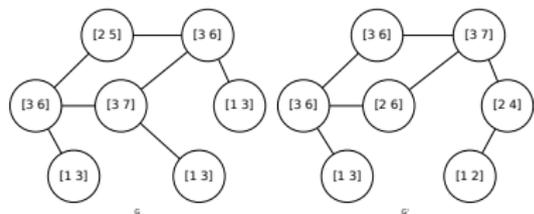


Data guided pyramid quantization.

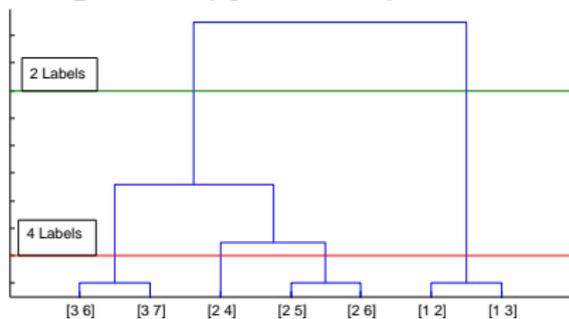


# A sequence of discretely labeled graphs

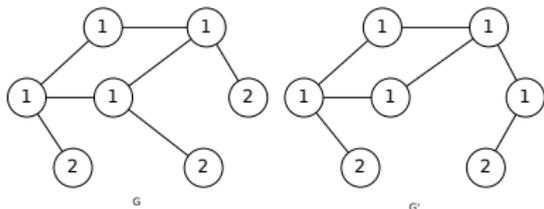
Given labeled graphs  $G$  and  $G'$



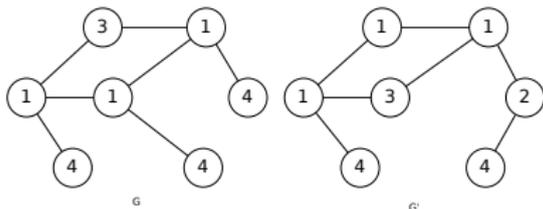
Data guided pyramid quantization.



Quantization level 1 with  $2^1$  number of labels.



Quantization level 2 with  $2^2$  number of labels



# Creating and combining subtree features

## Run Weisfeiler-Lehman on each quantization level

$$G = \left( G^{(0)}, \dots, G^{(L)} \right) \xrightarrow[\text{Lehman}]{\text{Weisfeiler}} \left( \phi_{(h)}^{(0)}(G^{(0)}), \dots, \phi_{(h)}^{(L)}(G^{(L)}) \right) = \widehat{\phi_{(h)}}(G)$$

where  $\phi_{(h)}^{(l)}(G^{(l)})$  are histograms of occurrences of the subtree patterns up to depth  $h$  at the quantization level  $l$  in graph  $G$

# Creating and combining subtree features

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## Learning to combine the quantization levels

- 1 Learn the selection of the subtree features  $\widehat{\phi_{(h)}}(G)$ .
- 2 Combine the subtree features  $\phi_{(h)}^{(l)}(G^{(l)})$  per level  $l$  into a kernel and then learn the combination of kernels.

# Learn the subtree patterns selection

- Labeled training data  $\{(\widehat{\phi_{(h)}}(G_i), y_i)\}_{1 \leq i \leq n} \in (\mathbb{N} \times \mathbb{R})^n$  where
  - $\widehat{\phi_{(h)}}(G_i)$  is the concatenation of histograms of the occurrences of subtree patterns up to depth  $h$  of graph  $G_i$  across all quantization levels,
  - $y_i$  is the ground truth label and

# Learn the subtree patterns selection

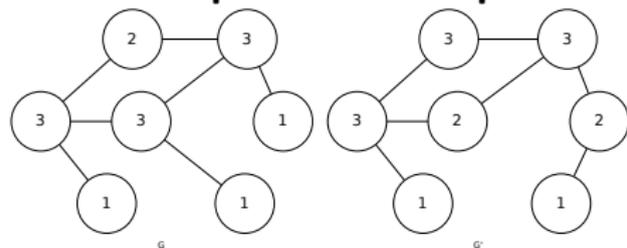
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  - $y_i$  is the ground truth label and
- Elastic Net [Zou 05b]

$$\arg \min_{w \in \mathbb{R}^d} \underbrace{\lambda_1 \|w\|_1}_{\ell_1 \text{ norm}} + \underbrace{\lambda_2 \|w\|_2^2}_{\ell_2 \text{ norm}} + \underbrace{\frac{1}{n} \sum_{i=1}^n \left( \langle w, \widehat{\phi_{(h)}}(G_i) \rangle - y_i \right)^2}_{\text{Squared loss}}$$

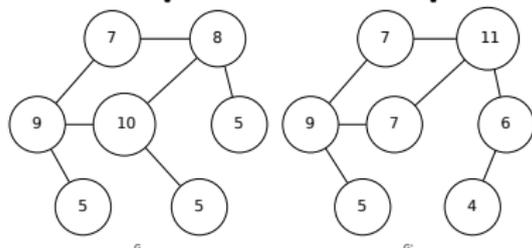
$\lambda_1, \lambda_2$  are scalar parameters controlling the degree of regularization.

# The intersection Weisfeiler-Lehman kernel

## Subtree patterns of depth 0.



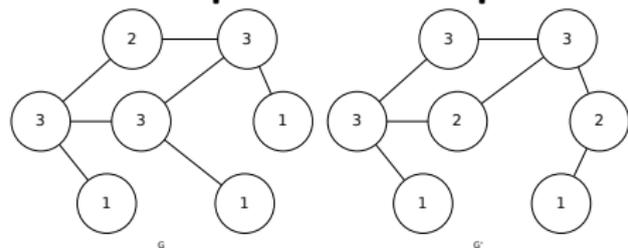
## Subtree patterns of depth 1.



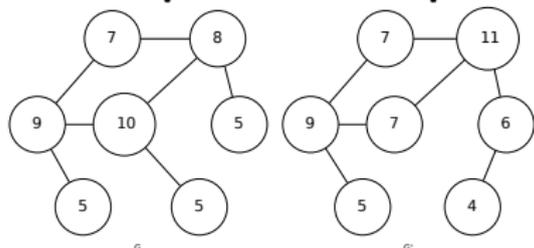
		Subtree patterns $h = 0$			Subtree patterns $h = 1$							
Labels $\{\Sigma_0, \Sigma_1\}$	=	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11
$\phi_{(1)}^{(l)}(G^{(l)})$	=	(3,	1,	3,	0,	3,	0,	1,	1,	1,	1,	0)
$\phi_{(1)}^{(l)}(G'^{(l)})$	=	(2,	2,	3,	1,	1,	1,	2,	0,	1,	0,	1)

# The intersection Weisfeiler-Lehman kernel

## Subtree patterns of depth 0.



## Subtree patterns of depth 1.



Subtree  
patterns  $h = 0$

Subtree patterns  $h = 1$

Labels $\{\Sigma_0, \Sigma_1\}$	=	{1, 2, 3,	4, 5, 6, 7, 8, 9, 10, 11}
$\phi_{(1)}^{(l)}(G^{(l)})$	=	(3, 1, 3,	0, 3, 0, 1, 1, 1, 1, 0)
$\phi_{(1)}^{(l)}(G'^{(l)})$	=	(2, 2, 3,	1, 1, 1, 2, 0, 1, 0, 1)
$\min(\phi_{(1)}^{(l)}(G^{(l)}), \phi_{(1)}^{(l)}(G'^{(l)}))$	=	(2, 1, 3,	0, 1, 0, 1, 0, 1, 0, 0)

The intersection Weisfeiler-Lehman kernel is defined :

$$k_{i-WLsubtree}^{(h)}(G^{(l)}, G'^{(l)}) = \sum_j^{|\Sigma_0 \cup \Sigma_1|} \min(\phi_{(1)}^{(l)}(G^{(l)}), \phi_{(1)}^{(l)}(G'^{(l)}))_j$$

# Multiple Kernel Learning

## Problem

For each pair of graphs  $G^{(l)}, G'^{(l)}$  for all the pyramid levels:

$$\left( K_{(h)}^{(0)}(G^{(0)}, G'^{(0)}), \dots, K_{(h)}^{(L)}(G^{(L)}, G'^{(L)}) \right)$$

we would like to learn a linear combination of them:

$$K_{(h)}(G, G') = \sum_{l=0}^L d_l K_{(h)}^{(l)}(G^{(l)}, G'^{(l)}), \text{ with } d_l \geq 0, \sum_{l=0}^L d_l = 1.$$

# Multiple Kernel Learning

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## Solutions

- Multiple kernel learning
- Average weighted kernel

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  - The Weisfeiler-Lehman algorithm
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- 3 Experiments
  - fMRI analysis problem
  - 3D shape classification
- 4 Other Problems
  - fMRI analysis and regularization methods
  - Neuromuscular disease classification
- 5 Conclusion

# fMRI Analysis

## Key information

- 1 Inherent spatial structure brains
- 2 Voxel activation is a continuous value

# fMRI Analysis

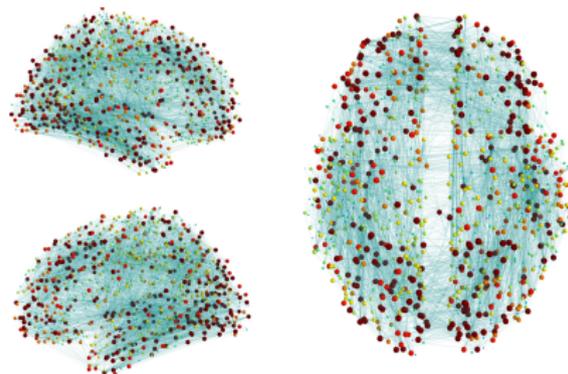
## Key information

- 1 Inherent spatial structure brains
- 2 Voxel activation is a continuous value



## Solution!

Represent fMRI as graphs with continuous labels.



# Dataset

## Cocaine Addiction Dataset

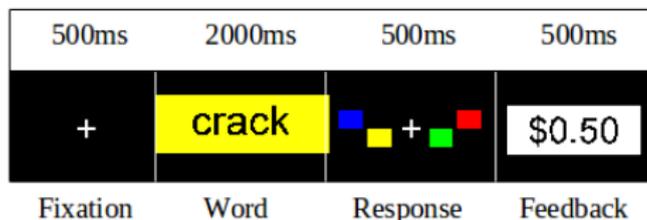
- 16 cocaine addicted vs 17 healthy subjects
- Drugstrop experiment with two varying conditions
  - the cue shown and
  - the monetary reward.

**Input** One contrast map per subject that is transformed into a graph.

**Objective** The classification between cocaine abuser and control group.

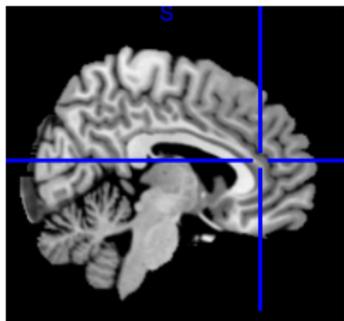
### Drugstrop Experiment

Total Stimulus Duration: 3.5s



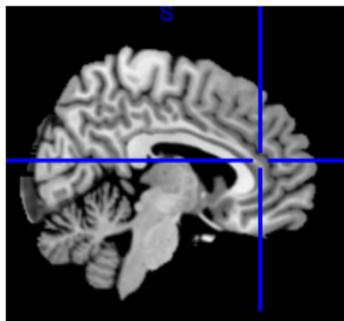
# Graph Transformation

Contrast map



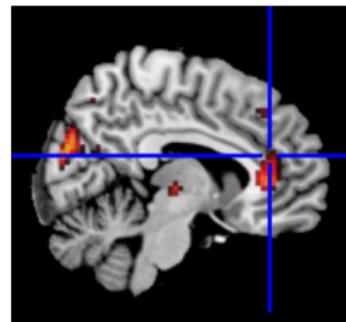
# Graph Transformation

Contrast map



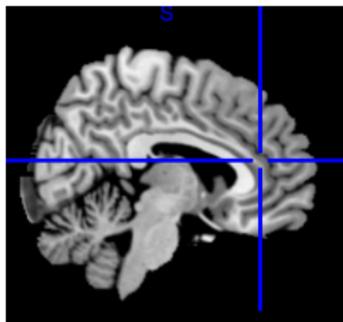
Elastic Net →

Selected voxels



# Graph Transformation

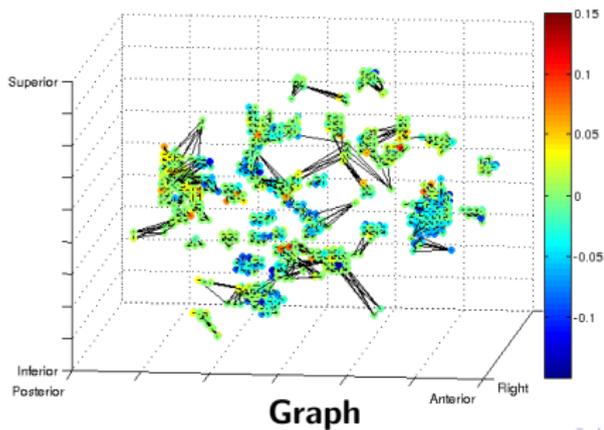
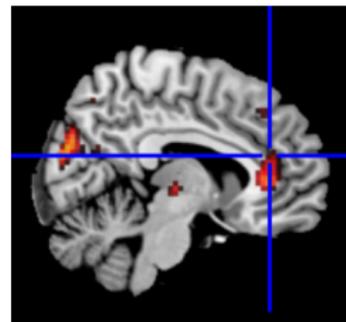
Contrast map



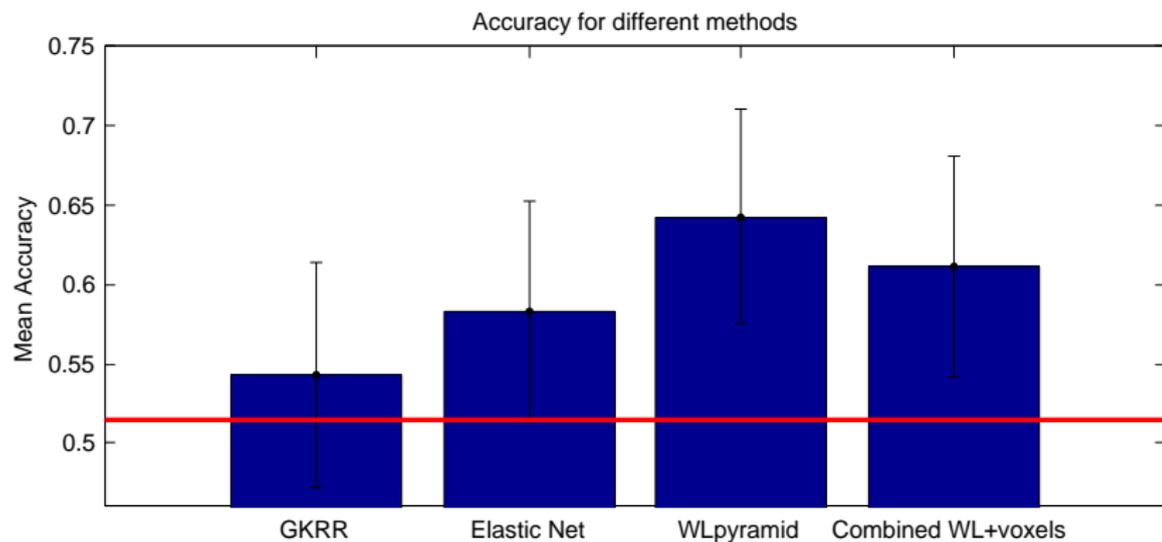
Elastic Net

knn

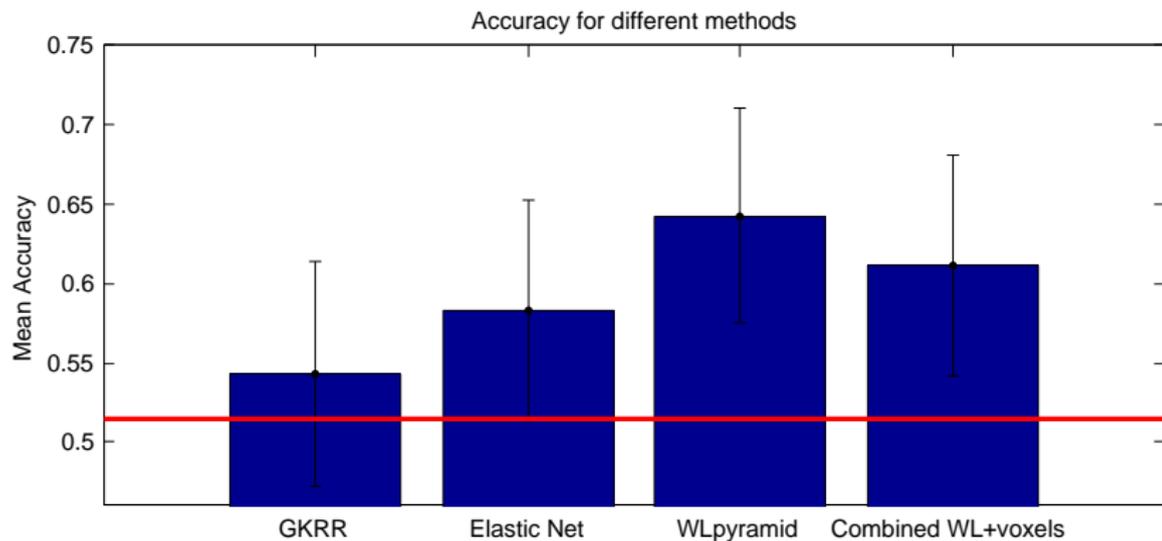
Selected voxels



# Performance



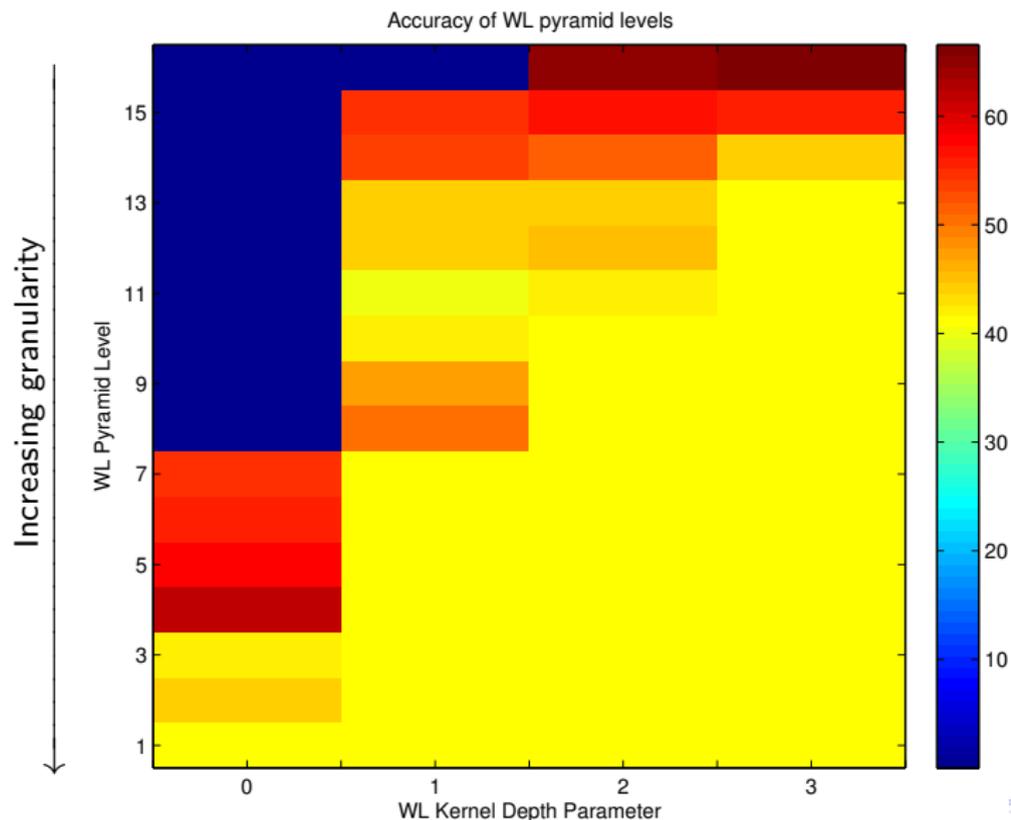
# Performance



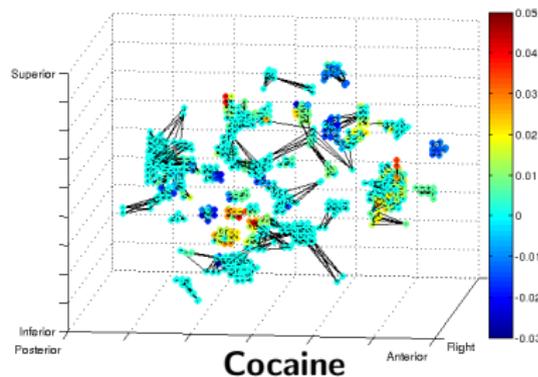
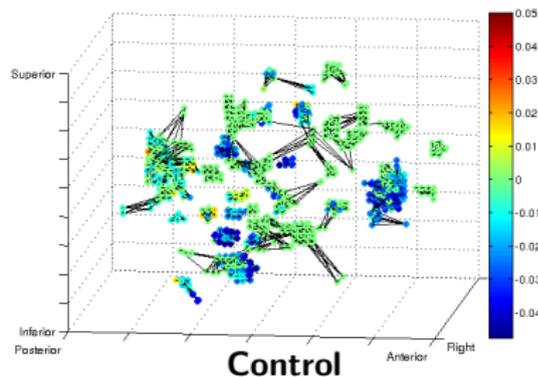
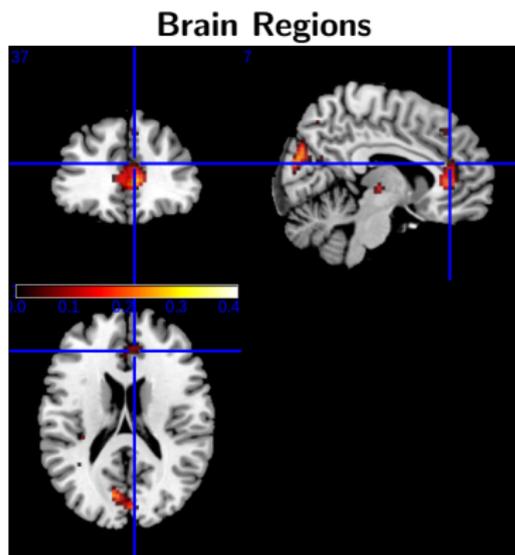
WLpyramid vs Elastic Net on raw voxels

Wilcoxon signed rank with  $p = 0.02$  show statistical significance.

# Performance per pyramid quantization level



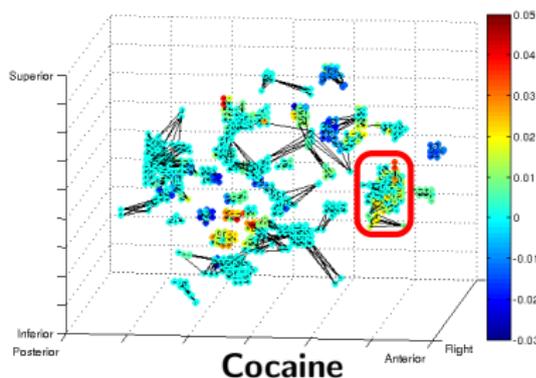
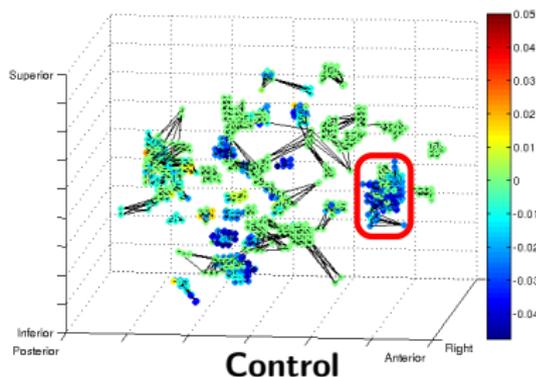
# Visualization of learned function



# Visualization of learned function

## Rostral Anterior Cingulate Cortex

- In cocaine addicted subjects deactivates during the drug Stroop experiment as compared to baseline.
- Its activity is normalized by oral methylphenidate where the dopamine transporters increase the extracellular dopamine, an increase which is associated with lower task-related impulsivity.



# 3D shape classification

## 3D shape problems

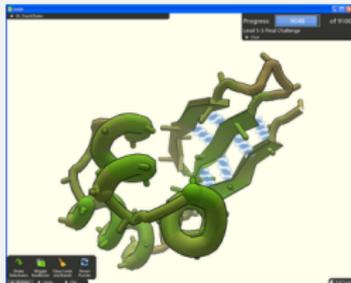
- Storage
- Classification
- Retrieval



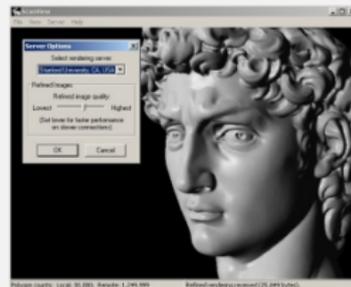
## Areas of applications



**3D Game**



**Chemoinformatics**

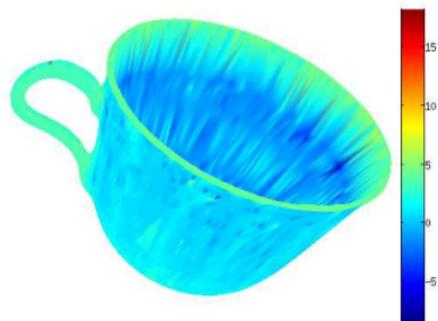


**Cultural heritage**

# 3D shape classification

## 3D shape problems

- Storage
- Classification
- Retrieval



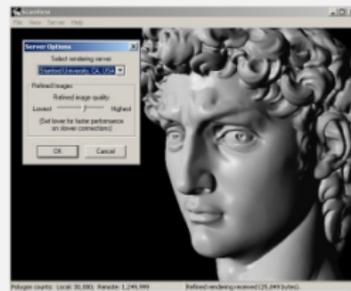
## Areas of applications



3D Game



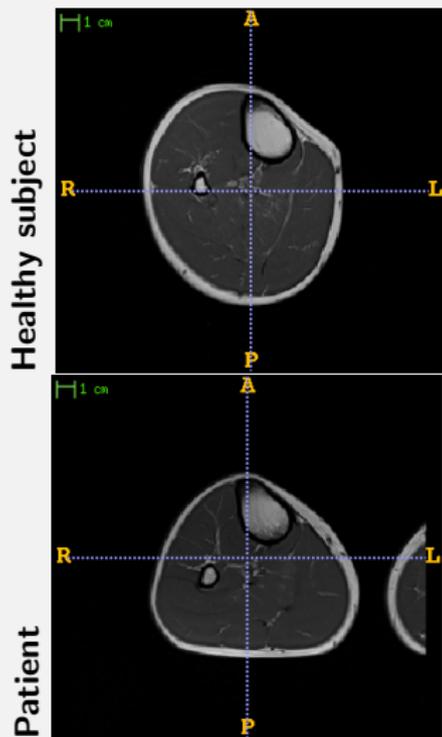
Chemoinformatics



Cultural heritage

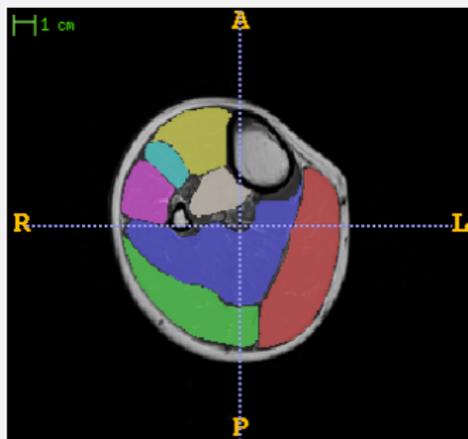
# 3D shape datasets

## Muscle Dataset



# 3D shape datasets

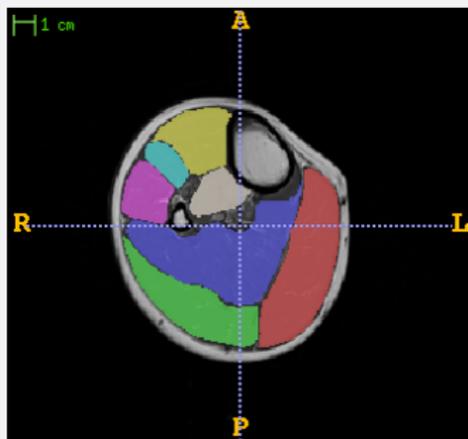
## Muscle Dataset



- 27 patients vs 14 healthy subjects
- MRI images of the calf muscles
- Segmented into 7 muscles

# 3D shape datasets

## Muscle Dataset



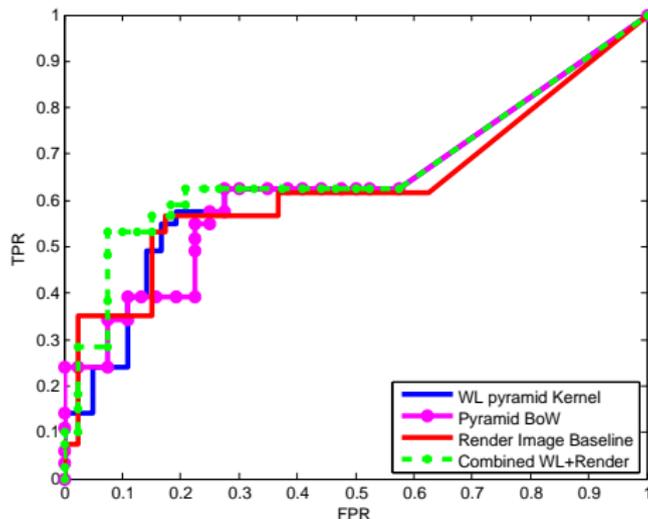
- 27 patients vs 14 healthy subjects
- MRI images of the calf muscles
- Segmented into 7 muscles

## SHREC 2013 Dataset



- 20 classes of generic objects, such as bed, biplane, mug, etc.
- Each class contains 18 models.
- In total 360 3D objects.

# Performance on the muscle dataset



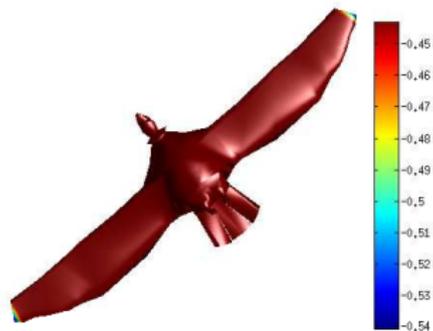
	WLpyramid Our Work	pyramid BoW	Rendering	Combined
<b>Accuracy</b>	78.00%	73.00%	75.50%	82.93%
<b>AUC</b>	0.6410	0.6361	0.6300	0.6648

## Performance on SHREC 2013

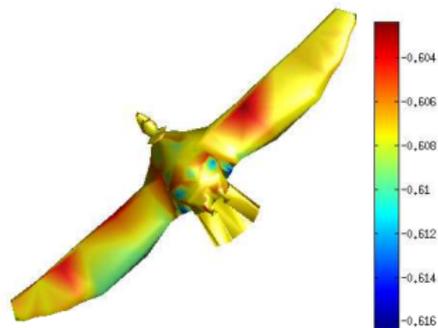
Class	WLpyramid Our Work	pyramid BoW	Rendering	Combined
Bird	0.85	0.83	0.85	<b>0.86</b>
Bicycle	0.84	0.87	0.90	0.90
Biped	0.89	0.88	0.99	0.99
Biplane	0.60	0.63	0.68	<b>0.69</b>
Bird	0.73	0.73	0.80	0.80
Bottle	0.76	0.76	0.79	<b>0.80</b>
Car	0.78	0.79	0.80	0.80
CellPhone	0.74	0.80	0.88	<b>0.89</b>
Chair	0.69	0.68	0.70	<b>0.72</b>
Cup	0.85	0.84	0.88	0.88
DeskLamp	0.80	0.80	0.88	<b>0.89</b>
Fish	1.00	1.00	1.00	1.00
Floorlamp	0.80	0.77	0.89	0.89
Insect	0.64	0.60	0.62	<b>0.66</b>
Monoplane	0.84	0.82	0.88	<b>0.90</b>
Mug	0.82	0.82	0.85	<b>0.87</b>
Phone	0.83	0.74	0.72	<b>0.83</b>
Quadruped	0.89	0.86	0.97	<b>0.98</b>
Sofa	0.76	0.75	0.74	<b>0.75</b>
Wheelchair	0.81	0.79	0.88	<b>0.90</b>
<b>Average</b>	0.80	0.79	0.84	<b>0.85</b>

# SHREC 2013 - Visualization of the learned weights

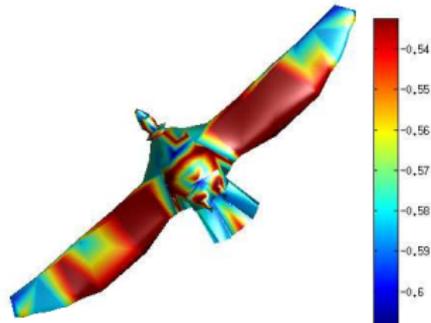
## Subtree patterns of depth 0



## Subtree patterns up to depth 2



## Subtree patterns up to depth 1



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# fMRI analysis and regularization methods

Regularizer	$\lambda\Omega(w)$
LASSO [Tibshirani 96]	$\lambda_1 \ w\ _1$

where

- $\lambda$  is a scalar controlling the degree of regularization,
- $|w|_i^\downarrow$  is the  $i$ th largest element of the vector  $|w|$ ,
- $k \in \{1, \dots, d\}$  is a scalar, user supplied parameter that correlates with the cardinality of  $w$  and
- $r$  is the unique integer in  $\{0, \dots, k - 1\}$  automatically selected by the algorithm.

## fMRI analysis and regularization methods

Regularizer	$\lambda\Omega(w)$
LASSO [Tibshirani 96]	$\lambda_1 \ w\ _1$
Elastic Net [Zou 05a]	$\lambda_1 \ w\ _1 + \lambda_2 \ w\ _2^2$

where

- $\lambda$  is a scalar controlling the degree of regularization,
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- $r$  is the unique integer in  $\{0, \dots, k - 1\}$  automatically selected by the algorithm.

## fMRI analysis and regularization methods

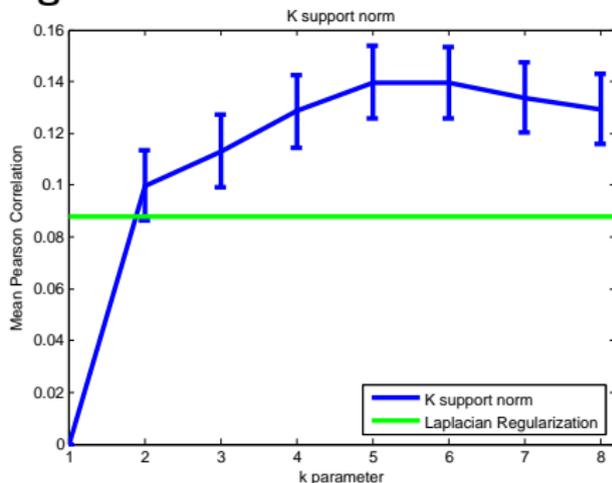
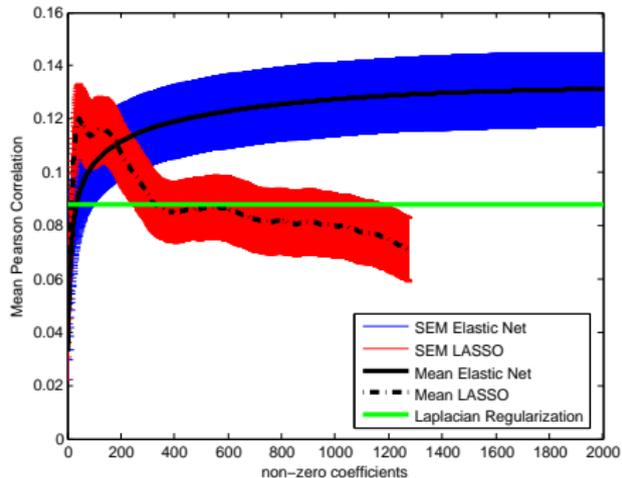
Regularizer	$\lambda\Omega(w)$
LASSO [Tibshirani 96]	$\lambda_1 \ w\ _1$
Elastic Net [Zou 05a]	$\lambda_1 \ w\ _1 + \lambda_2 \ w\ _2^2$
$k$ -support norm [Argyriou 12]	$\lambda \left( \overbrace{\sum_{i=1}^{k-r-1} ( w _i^\downarrow)^2}^{\ell_2 \text{ norm}} + \frac{1}{r+1} \left( \overbrace{\sum_{i=k-r}^d  w _i^\downarrow}^{\ell_1 \text{ norm}} \right)^2 \right)^{\frac{1}{2}}$

where

- $\lambda$  is a scalar controlling the degree of regularization,
- $|w|_i^\downarrow$  is the  $i$ th largest element of the vector  $|w|$ ,
- $k \in \{1, \dots, d\}$  is a scalar, user supplied parameter that correlates with the cardinality of  $w$  and
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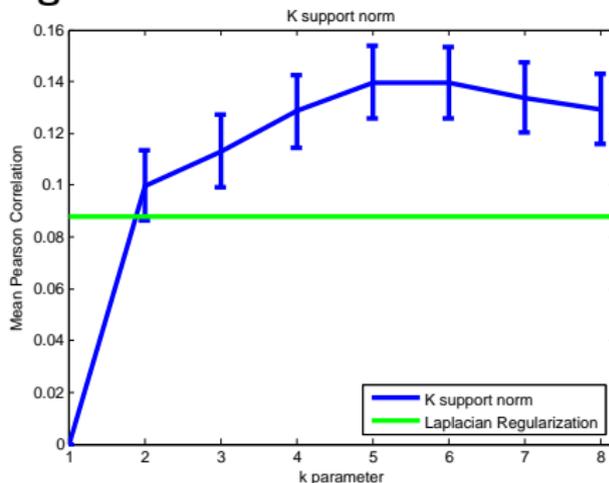
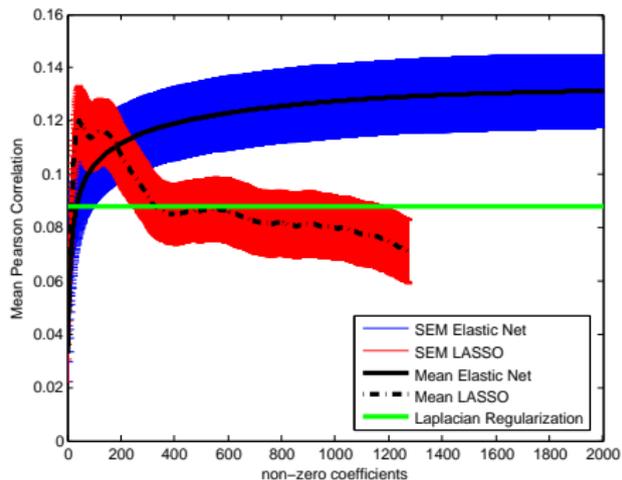
# Quantitative Results

## Picture viewing dataset



# Quantitative Results

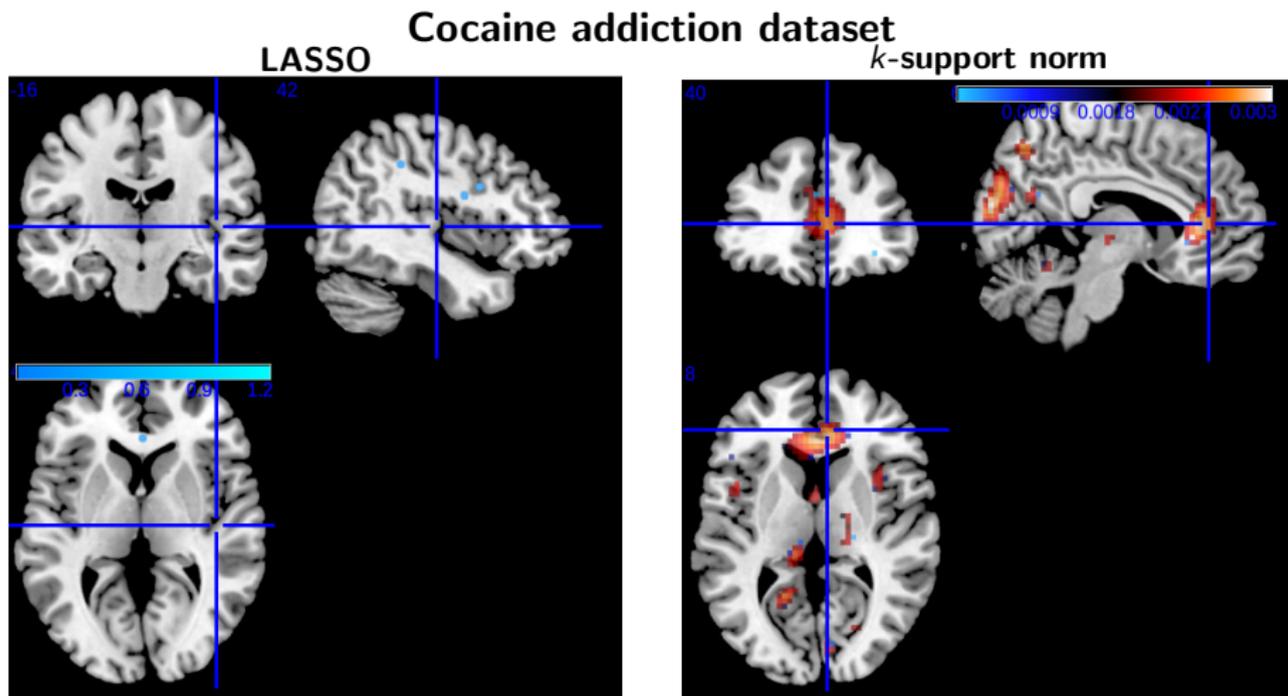
## Picture viewing dataset



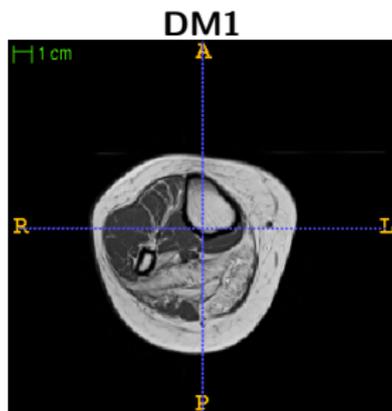
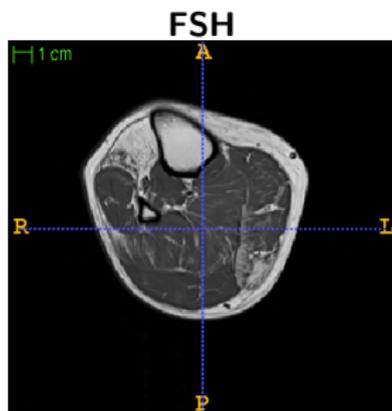
*k*-support norm vs LASSO and Elastic Net

Wilcoxon signed rank test with  $p = 0.05$  show statistical significance.

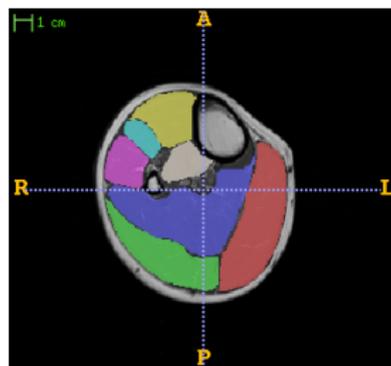
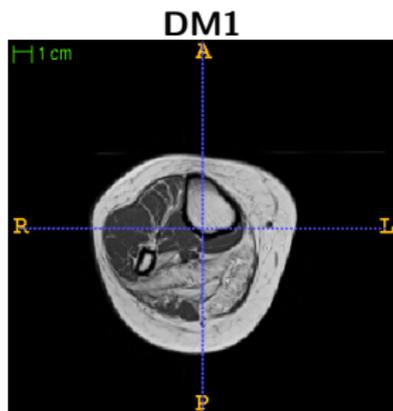
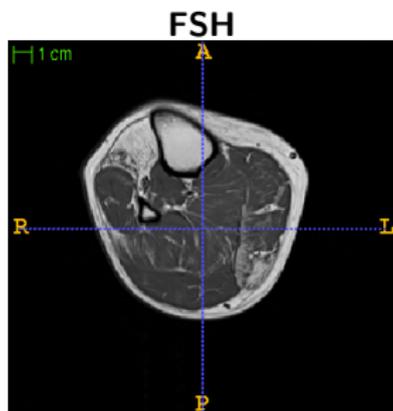
# Qualitative Results



# Neuromuscular disease classification



# Neuromuscular disease classification



- the mean T1/T2 signal,
- the Signal to Noise Ratio,
- the Fractional Anisotropy,
- the trace of the diffusion tensor,
- the volume of the tensor, etc.

## $k$ -support regularized SVM

The Support Vector Machine is defined as the following optimization problem:

$$\begin{aligned} \min_{w \in \mathbb{R}^d, b \in \mathbb{R}, \xi \in \mathbb{R}^n} \quad & \lambda \|w\|_2^2 + \sum_{i=1}^n \xi_i \\ \text{s.t.} \quad & y_i (\langle w, x_i \rangle + b) \geq 1 - \xi_i, \quad \xi_i \geq 0, \quad \forall i. \end{aligned}$$

where

- $\lambda$  is a scalar, user supplied parameter controlling the degree of regularization,

## $k$ -support regularized SVM

The  $k$ -support norm regularized SVM ( $k$ sup-SVM) is defined as the following optimization problem:

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where

- $\lambda$  is a scalar, user supplied parameter controlling the degree of regularization,
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- $\|w\|_k^{sp}$  is the  $k$ -support penalty.

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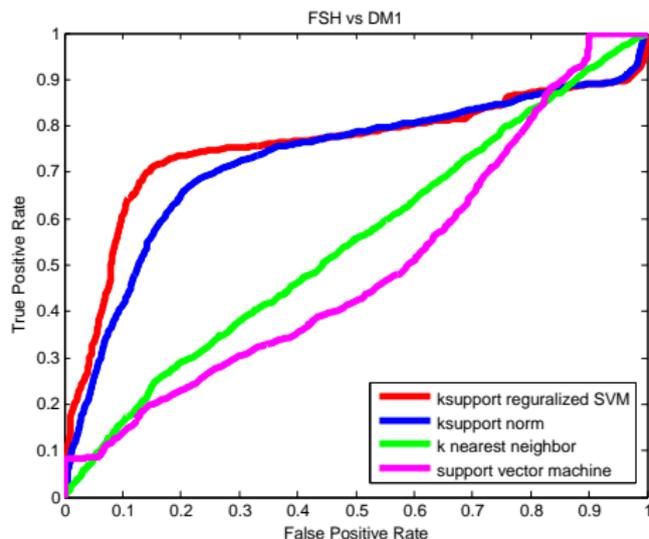
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### Advantage

Solution is sparse but correlated subset of discriminative variables.

# Performance

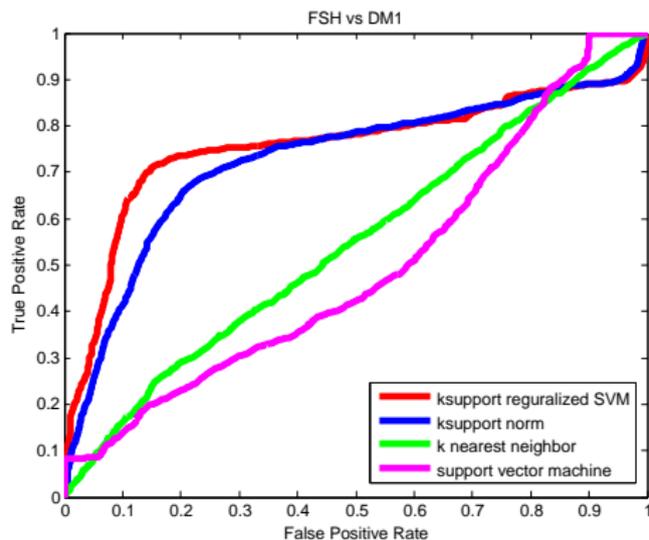


Method	AUC
<i>ksup</i> -SVM - Our work	0.756
<i>k</i> -support with SL	0.726
<i>knn</i>	0.537
SVM	0.494

Method	Accuracy
<i>ksup</i> -SVM - Our work	77 ± 0.013
<i>k</i> -support with SL	74 ± 0.006
<i>knn</i>	61 ± 0.015
SVM	59 ± 0.015

Chance is 60%.

# Performance



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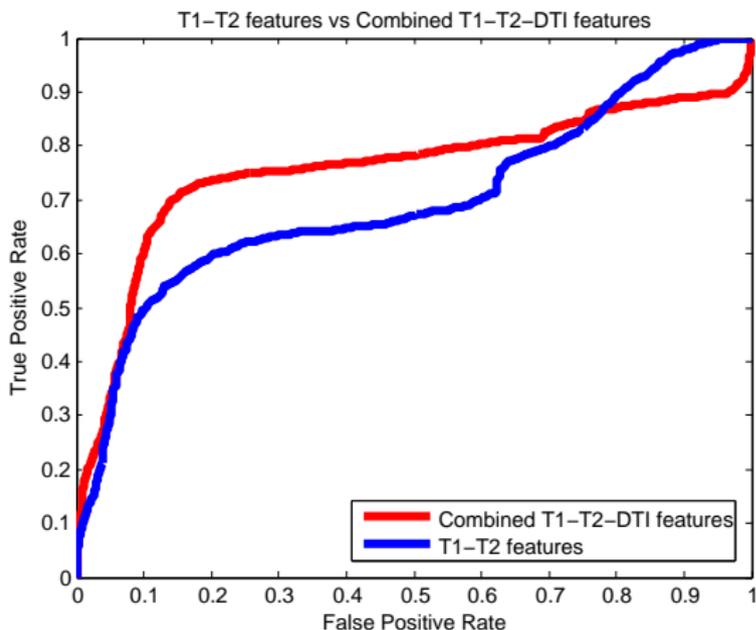
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*ksup*-SVM vs the rest methods

Wilcoxon signed rank test with  $p \ll 10^{-9}$  show statistical significance.

# Structured and DTI features vs Structured features only

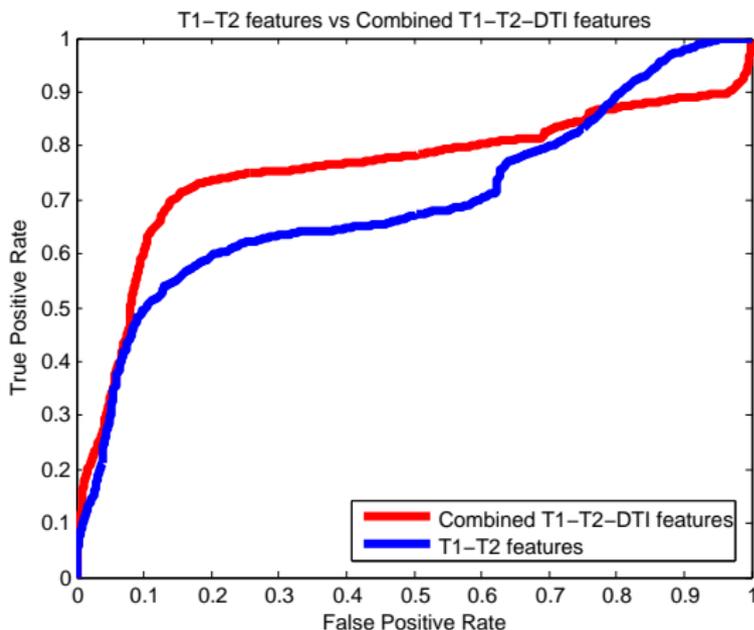


Features Used	AUC
MRI and DTI	0.756
MRI only	0.697

Features Used	Accuracy
MRI and DTI	$77 \pm 0.013$
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# Structured and DTI features vs Structured features only



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# Table of Content

- 1 Introduction to Graphs
- 2 The pyramid quantized Weisfeiler-Lehman graph representation
  - Overview
  - The Weisfeiler-Lehman algorithm
  - The pyramid quantization strategy
  - A sequence of discretely labeled graphs
  - Learning the combination of the pyramid levels.
- 3 Experiments
  - fMRI analysis problem
  - 3D shape classification
- 4 Other Problems
  - fMRI analysis and regularization methods
  - Neuromuscular disease classification
- 5 Conclusion

## Contributions - Methodological

### The *pyramid quantized Weisfeiler-Lehman graph representation*

- A novel algorithm for comparing graphs with vector labels.

### *k*-support regularized SVM

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- Linear computation time in the number of graphs, in the number of edges in the graphs and in the depth of subtree patterns.

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- A novel regularized SVM algorithm.
- Correlated sparse solution under the SVM framework.

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### *k-support regularized SVM*

- A novel regularized SVM algorithm.
- Correlated sparse solution under the SVM framework.
- Evaluation on a neuromuscular disease task.

# Contributions

## Methodological

Code from both algorithms is available online under GNU-GPL at <http://cvc.centrale-ponts.fr/personnel/gkirtzou/code>

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- In the fMRI analysis, we saw that the interconnections between voxels can contain additional information about brain structure.
- In the neuromuscular dystrophy classification task, we saw that features extracted from DTI images provide significant information.
- Interpretation of 3D shape meshes as annotated graphs.

# Future Work

## Medical image analysis

- Evaluation of  $k$ -support norm regularization on fMRI analysis problem in larger scale.
- Evaluation of  $k$ -support regularized SVM on neuromuscular disease discrimination in larger scale.
- Exploration of different constructions of the graphs from fMRI.

# Future Work

## Medical image analysis

- Evaluation of  $k$ -support norm regularization on fMRI analysis problem in larger scale.
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- Exploration of different constructions of the graphs from fMRI.

## Graph kernels

- Comparison on partially matching subtree patterns.
- Comparison on partially labeled graphs.

# Publications



K. Gkirtzou, and M. Blaschko

*The pyramid quantization Weisfeiler-Lehman graph representation*

Submitted to Pattern Recognition



K. Gkirtzou, J. Honorio, D. Samaras, R. Goldstein and M. Blaschko

*FMRI analysis of cocaine addiction using k-support sparsity*

In International Symposium on Biomedical Imaging 2013

Oral Presentation - 19% acceptance rate



K. Gkirtzou, DJ. Francois, G. Bassez, A. Sotiras, A.Rahmouni, T. Varacca,

N. Paragios and M. Blaschko

*Sparse classification with MRI based markers for neuromuscular disease classification.*

In LNCS series of Machine Learning in Medical Imaging 2013

Oral Presentation - 26% acceptance rate



K. Gkirtzou, J. Honorio, D. Samaras, R. Goldstein and M. Blaschko

*fMRI Analysis with Sparse Weisfeiler-Lehman Graph Statistics*

In LNCS series of Machine Learning in Medical Imaging 2013

Poster Presentation - 56% acceptance rate

# Table of Contents - Appendix

## 6 Extra slides

## 7 ISBI 2013

- Introduction
- Methods
- Results
- Conclusions - Future Work

## 8 MLMI 2013

- Introduction
- Materials and Methods
- Results
- Discussion

# Table of Content

## 6 Extra slides

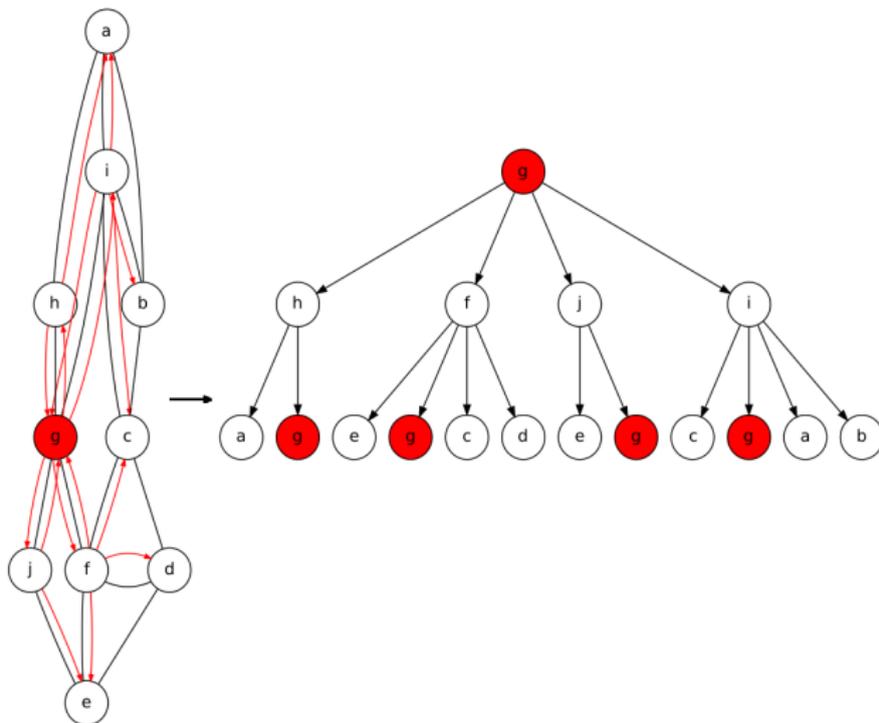
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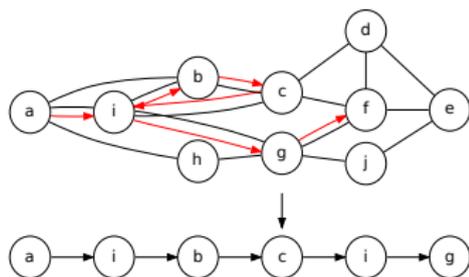
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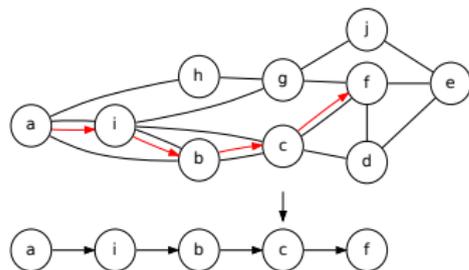
# Subtree patterns



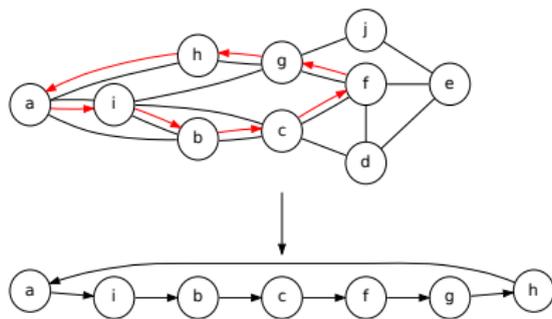
# Walk, path and cycle on graph



(a) Graph Walk



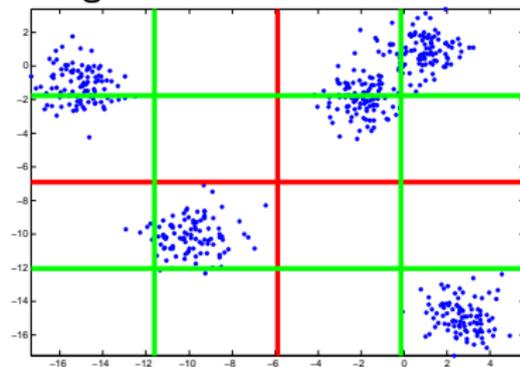
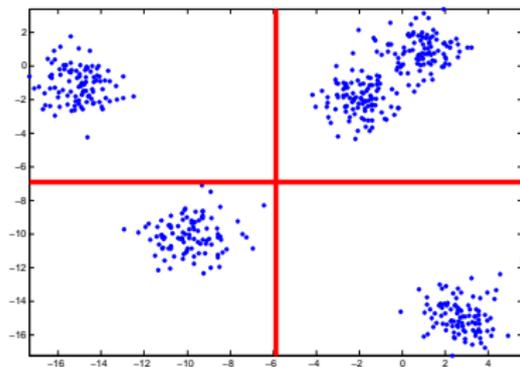
(b) Graph Path



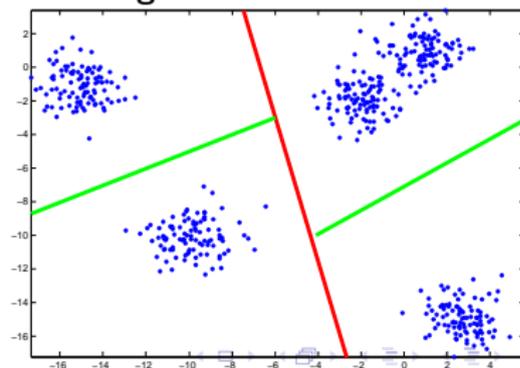
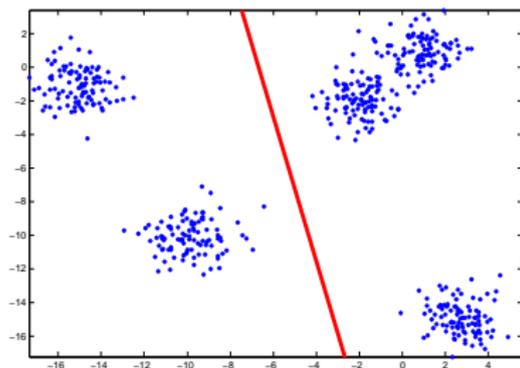
(c) Graph Cycle

# Binning strategies

## Fixed binning



## Data guided binning



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# fMRI Analysis of Cocaine Addiction using $k$ -support sparsity

Exploring regularization techniques for fMRI analysis

K. Gkirtzou<sup>1,2</sup>, Jean Honorio<sup>3</sup>, Dimitris Samaras<sup>1,3</sup>, Rita Goldstein<sup>4</sup>,  
Matthew B. Blaschko<sup>1,2</sup>



10th April, 2013

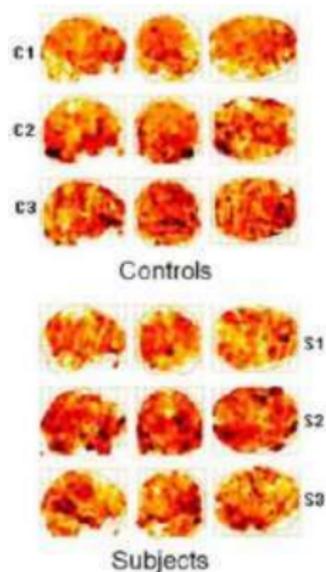
# fMRI Analysis

## Goal of fMRI analysis

The goal of fMRI data analysis is to detect correlations between brain activation and a task the subject performs during the scan.

## Common problems in fMRI analysis

- high-dimensional space
- small number of samples
- high levels of noise



# Related Work

## Previous Related Work

- Generalized Linear Model
- Support vector machines [Song 11]
- Kernel canonical correlation analysis [Hardoon 07, Blaschko 09, Blaschko 11].
- Independent component analysis [Bartels 04, Bartels 05]
- Regression models (OLS, Ridge Regression, LASSO, Elastic Net) [Carroll 09, Ng 12].

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## Regularized methods explored in this work

- LASSO [Tibshirani 96]
- Elastic Net [Zou 05a]
- $k$ -support norm [Argyriou 12]

# Sparsity regularization - Mathematical framework

- Labeled training data  $\{(x_1, y_1), \dots, (x_n, y_n)\} \in (\mathbb{R}^d \times \mathbb{R})^n$ 
  - $x_i$  is the output of a fMRI scan
  - $y_i$  is the ground truth label
- Loss function

$$\arg \min_{w \in \mathbb{R}^d} \lambda \Omega(w) + \frac{1}{n} \sum_{i=1}^n (\langle w, x_i \rangle - y_i)^2$$

- $\lambda$  is a scalar parameter controlling the degree of regularization
- $\Omega$  is a scalar valued function monotonic in a norm of  $w$ .

# Sparsity regularization - Mathematical framework

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## Penalty function

Regularizer	$\lambda \Omega(w)$
LASSO [Tibshirani 96]	$\lambda_1 \ w\ _1$
Elastic net [Zou 05a]	$\lambda_1 \ w\ _1 + \lambda_2 \ w\ _2^2$

## $k$ support norm - Penalty function

The  $k$ -support norm [Argyriou 12] can be computed as

$$\lambda\Omega(w) = \lambda\|w\|_k^{sp} = \lambda \left( \overbrace{\sum_{i=1}^{k-r-1} (|w|_i^\downarrow)^2}^{\ell_2 \text{ norm}} + \frac{1}{r+1} \overbrace{\left( \sum_{i=k-r}^d |w|_i^\downarrow \right)^2}^{\ell_1 \text{ norm}} \right)^{\frac{1}{2}}$$

where

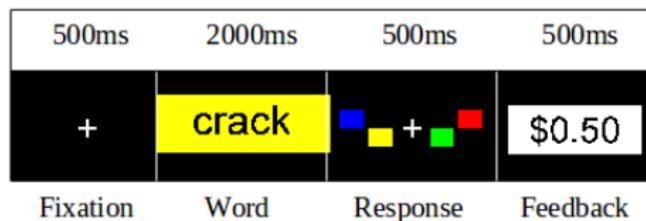
- $\lambda$  is a scalar, user supplied parameter controlling the degree of regularization,
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- $r$  is the unique integer in  $\{0, \dots, k-1\}$  automatically selected by the algorithm.

# Dataset 1

## Cocaine Addiction Dataset

- 16 cocaine addicted vs 17 control subjects
- Drugstrop experiment with two varying conditions
  - the cue shown
  - the monetary reward
- Using one contrast map per subject
- The discriminative task is the classification between cocaine abuser and control group

Total Stimulus Duration: 3.5s



Drugstrop Experiment

# Dataset 2

## Natural viewing dataset

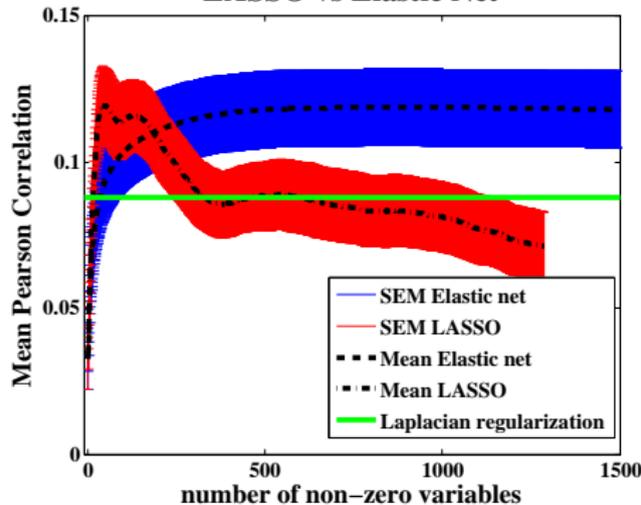
- A healthy subject in a free-viewing setting
- Using the whole time series
- The discriminative task is the prediction of a “Temporal Contrast” variable

## Experimental setup

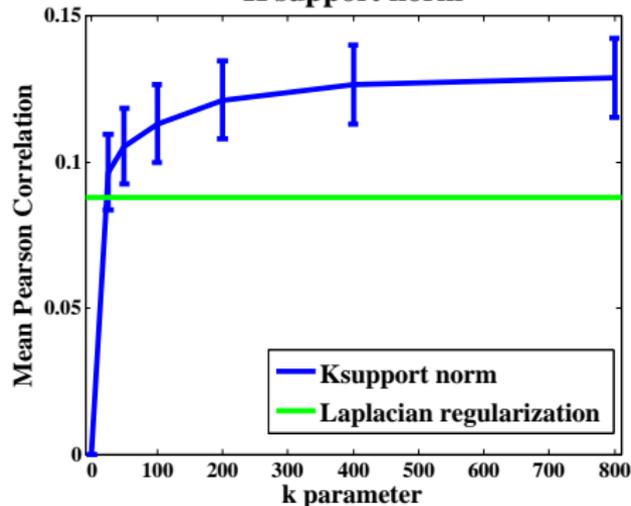
- 100 random permutations trials
- Training on the 80% of data
- Testing on the rest 20% of data

# Quantitative results

## LASSO vs Elastic Net



## K support norm

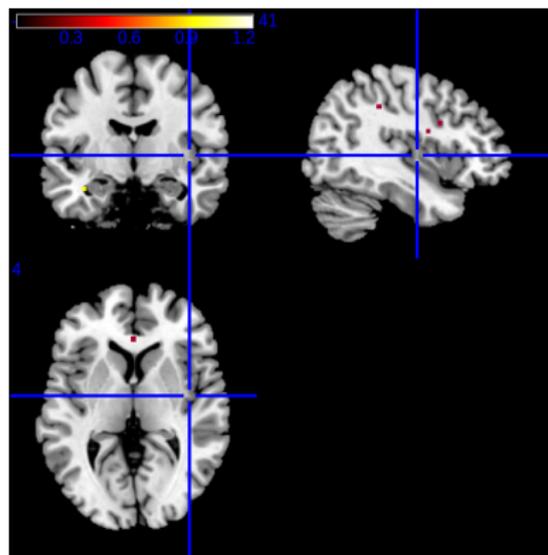


Natural viewing dataset

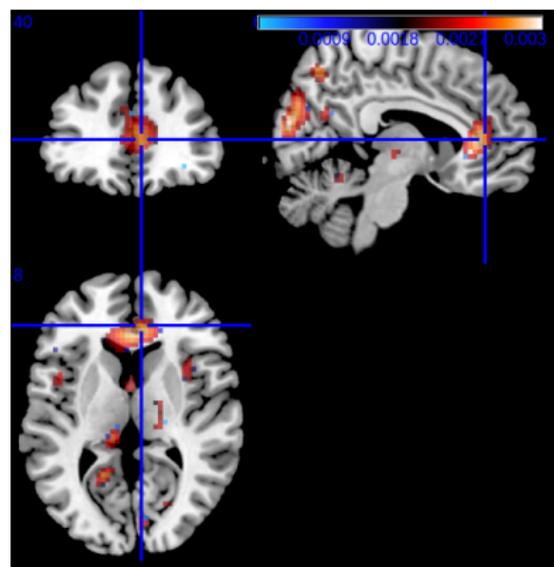
Elastic Net vs  $k$ -support norm

Wilcoxon signed rank test with  $p$ -value  $\ll 0.05$

# Qualitative results

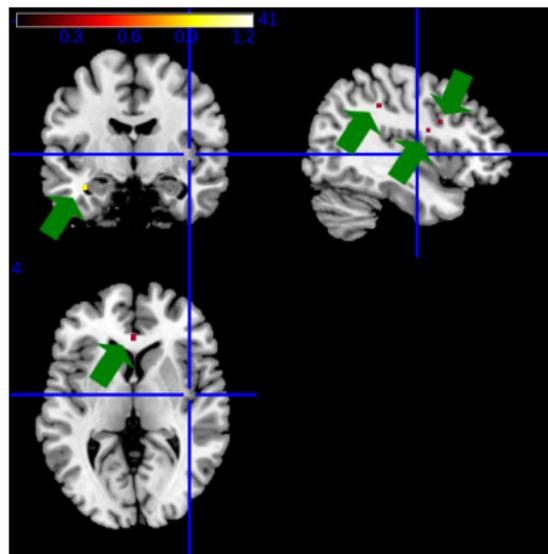


LASSO

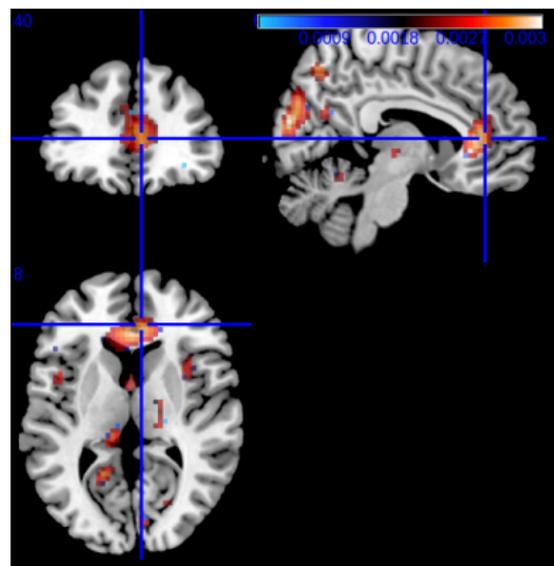
 $k$ -support norm

Cocaine addiction dataset

# Qualitative results



LASSO

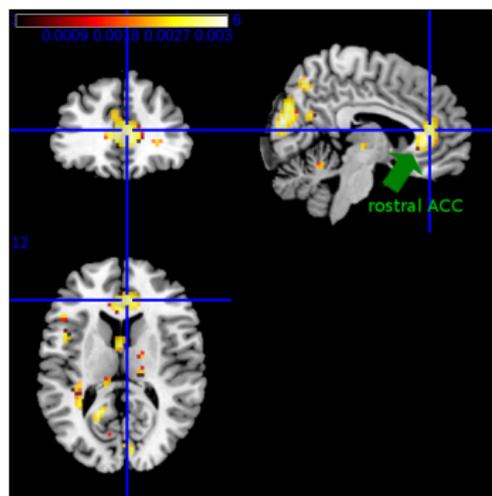
 $k$ -support norm

Cocaine addiction dataset

# Rostral Anterior Cingulate Cortex

## Rostral Anterior Cingulate Cortex

- In cocaine addicted subjects deactivates during the drug Stroop experiment as compared to baseline.
- Its activity is normalized by oral methylphenidate where the dopamine transporters increase the extracellular dopamine, an increase which is associated with lower task-related impulsivity.
- In cigarette smokers was responsive to pharmacotherapeutic interventions.
- In depression may be a marker of treatment response.



*k*-support norm

# Conclusions - Future Work

## Conclusions

- The  $k$ -support norm can boost the predictive performance of the LASSO and elastic net.
- The LASSO does not show a meaningful sparsity pattern.
- The brain regions implicated in addiction by the  $k$ -support norm coincide with previous results on addiction.
- Code available online ::  
<http://www.centrale-ponts.fr/personnel/gkirtzou/code/>

# Conclusions - Future Work

## Conclusions

- The  $k$ -support norm can boost the predictive performance of the LASSO and elastic net.
- The LASSO does not show a meaningful sparsity pattern.
- The brain regions implicated in addiction by the  $k$ -support norm coincide with previous results on addiction.
- Code available online ::  
<http://www.centrale-ponts.fr/personnel/gkirtzou/code/>

## Future Work

- Exploring the structural information of the brain

# Questions

## Acknowledgements

- This work is partially funded by
  - the European Research Council under the Seventh Framework Programme (FP7/2007-2013)/ERC Grant 259112.
  - NIDA R21 DA034954 under the SUBSample project from the DIGITEO Institute, France.
  - “Machine learning discovery of patterns of self-regulation in drug addiction and intermittent explosive disorder” NIA 1R21DA034954-01
- We thank A. Bartels for providing data.



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- Discussion

# Sparse classification with MRI based markers for neuromuscular disease categorization

Exploring regularization techniques for fMRI analysis

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Aristeidis Sotiras<sup>4</sup>, Alain Rahmouni<sup>3</sup>, Thibault Varacca<sup>3</sup>, Nikos  
Paragios<sup>1,2</sup>, Matthew B. Blaschko<sup>1,2</sup>



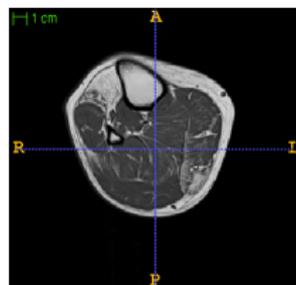
22th September, 2013

# Neuromuscular Diseases

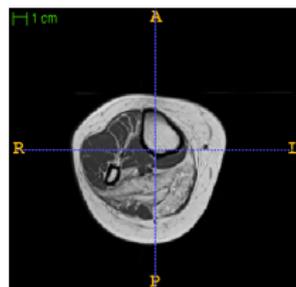
## Problem

- Myopathies are neuromuscular diseases that result functional anomalies including
  - fat infiltration
  - atrophy
  - weakness of the muscle and
  - paralysis.
- In this study, we focus on categorization patients between Facioscapulohumeral muscular dystrophy (FSH) and myotonic muscular dystrophy type 1 (DM1) using MRI based markers.

T1-weighted MR images of the calf.



(d) FSH



(e) DM1

# Our approach and Related Work

## Our approach

- 1 Features extracted from both structured MR Imaging (T1 and T2 weighted) and Diffusion Tensor Imaging
- 2 a novel structured sparsity algorithm, the  $k$ -support regularized SVM ( $k$ sup-SVM)

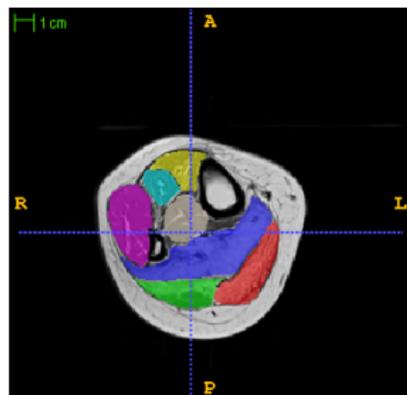
## Related Work

- DTI on Neuroimaging studies
  - Alzheimer's disease [Klöppel 08]
  - male-female or older-younger classification [Lao 04]
  - temporal classification of block design fMRI data [LaConte 05]
  - the study of autism spectrum disorder [Ingalhalikar 11]
- DTI on different clinical scenarios
  - the human tongue [Gilbert 05]
  - the heart muscle [Gilbert 05]
  - the human calf muscle [Galban 04]

# Data description

## Dataset

- 25 subjects, 10 affected by FSH and 15 affected by DM1.
- T1-weighted, T2-weighted and Diffusion Tensor Images of the calf muscle.
- Obtained volumes  $64 \times 64 \times 20$  voxels with voxel resolution  $3.125\text{mm} \times 3.125\text{mm} \times 7\text{mm}$



Color	Muscle
Yellow	the anterior tibialis
Cyan	the extensor digitorum longus
Magenta	the peroneus longus
White	the posterior tibialis
Blue	the soleus
Green	the lateral gastrocnemius
Red	the medial gastrocnemius

## Structural and DTI features

We extract for every muscle the following features from the structural data:

- 1 the absolute volume,
- 2 the mean T1 signal,
- 3 the mean T2 signal, and
- 4 the Signal to Noise Ration (SNR).

and from the DTI data:

- 1 the Fractional Anisotropy (FA),
- 2 the trace of the diffusion tensor,
- 3 the volume of the tensor,
- 4 the eigenvalues (L1, L2, L3),
- 5 the planar coefficient (Cp), and
- 6 the linear coefficient (Cl).

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- 6 the linear coefficient (Cl).

### Number of Features

4 Structured features  $\times$  8 DTI features  $\times$  7 muscles = 84 features

## $k$ support norm - Regularization term

The  $k$ -support norm [Andreas Argyriou 12] can be computed as

$$\lambda \Omega(w) = \lambda \|w\|_k^{sp} = \lambda \left( \overbrace{\sum_{i=1}^{k-r-1} (|w|_i^\downarrow)^2}^{\ell_2 \text{ norm}} + \frac{1}{r+1} \overbrace{\left( \sum_{i=k-r}^d |w|_i^\downarrow \right)^2}^{\ell_1 \text{ norm}} \right)^{\frac{1}{2}}$$

where

- $\lambda$  is a scalar, user supplied parameter controlling the degree of regularization,
- $|w|_i^\downarrow$  is the  $i$ th largest element of the vector  $|w|$ ,
- $k \in \{1, \dots, d\}$  is a scalar, user supplied parameter that negative correlates with the cardinality of  $w$  and
- $r$  is the unique integer in  $\{0, \dots, k-1\}$  automatically selected by the algorithm.

## $k$ -support regularized SVM

The Support Vector Machine is defined as the following optimization problem:

$$\begin{aligned} \min_{w \in \mathbb{R}^d, b \in \mathbb{R}, \xi \in \mathbb{R}^n} \quad & \lambda \|w\|_2^2 + \sum_{i=1}^n \xi_i \\ \text{s.t.} \quad & y_i (\langle w, x_i \rangle + b) \geq 1 - \xi_i, \quad \xi_i \geq 0, \quad \forall i. \end{aligned}$$

where

- $\lambda$  is a scalar, user supplied parameter controlling the degree of regularization,

## $k$ -support regularized SVM

The  $k$ -support norm regularized SVM ( $k$ sup-SVM) is defined as the following optimization problem:

$$\begin{aligned} \min_{w \in \mathbb{R}^d, b \in \mathbb{R}, \xi \in \mathbb{R}^n} \quad & \lambda \|w\|_k^{sp} + \sum_{i=1}^n \xi_i \\ \text{s.t.} \quad & y_i (\langle w, x_i \rangle + b) \geq 1 - \xi_i, \quad \xi_i \geq 0, \quad \forall i. \end{aligned}$$

where

- $\lambda$  is a scalar, user supplied parameter controlling the degree of regularization,
- $k \in \{1, \dots, d\}$  is a scalar, user supplied parameter that negative correlates with the cardinality of  $w$  and
- $\|w\|_k^{sp}$  is the  $k$ -support penalty.

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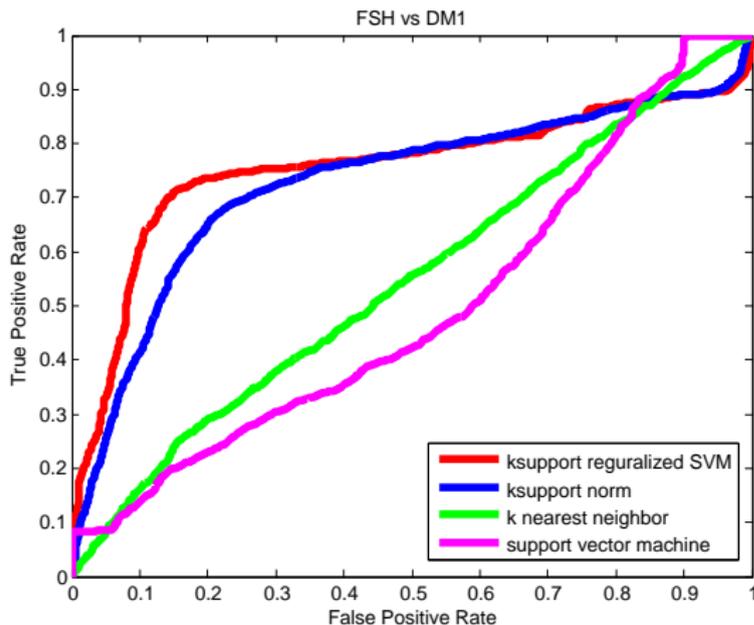
### Advantage

Solution is sparse but correlated subset of discriminative variables.

# Experimental Setting

- Methods under comparison:
  - $k$ sup-SVM with  $k \in \{1, 10, 20, 40, 80\}$  and  $\lambda \in \{1, 10, 1000\}$ .
  - knn with  $k \in \{1, 3, 5, 7, 10\}$
  - SVM with kernel functions
    - linear,
    - polynomial of third degree, and
    - radial basis function (RBF)with a soft-margin parameter  $C \in \{10^{-3}, 10^0, 10^3\}$ .
  - $k$ -support norm with  $k \in \{1, 10, 20, 40, 80\}$  and  $\lambda \in \{1, 10, 1000\}$ .
- 1000 trials of random split, with 80% of the data used for training and the rest 20% for testing.

# Performance

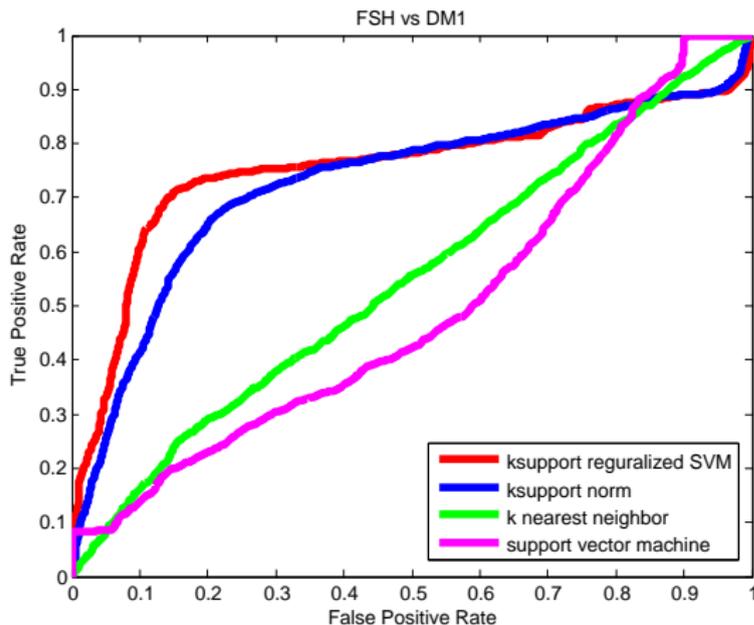


Method	AUC
<i>ksup</i> -SVM	0.756
<i>k</i> -support norm	0.726
<i>knn</i>	0.537
SVM	0.494

Method	Accuracy
<i>ksup</i> -SVM	77 ± 0.013
<i>k</i> -support norm	74 ± 0.006
<i>knn</i>	61 ± 0.015
SVM	59 ± 0.015

Chance is 60%.

# Performance



Method	AUC
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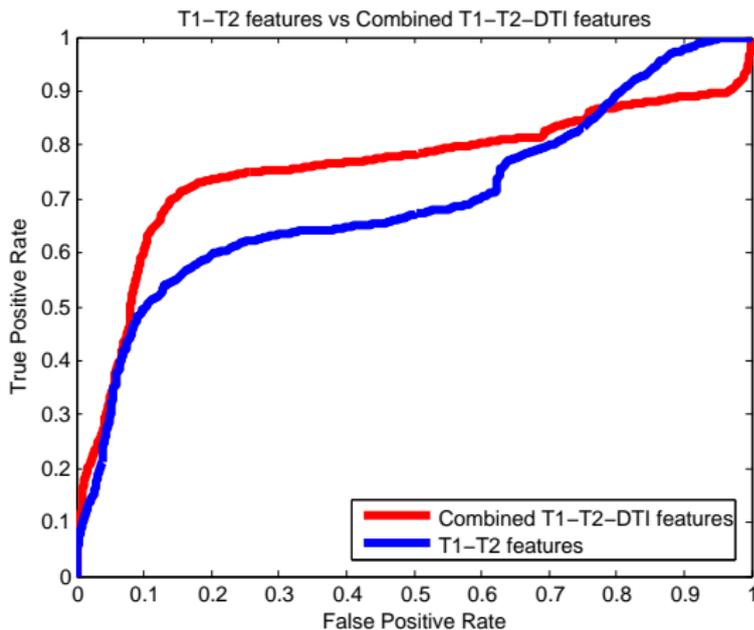
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Chance is 60%.

*ksup*-SVM vs rest methods

Wilcoxon signed rank test with  $p\text{-value} \ll 10^{-9}$

# Structured and DTI features vs Structured features only



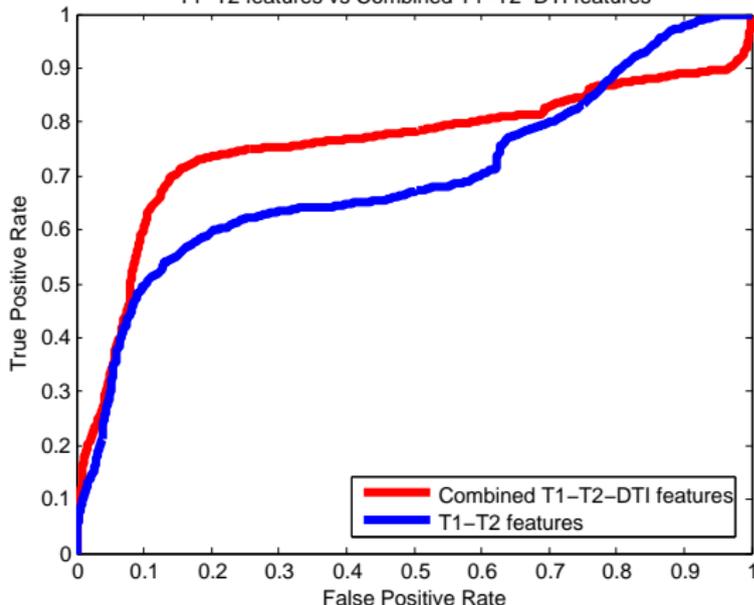
Features Used	AUC
T1, T2 and DTI	0.756
T1 and T2	0.697

Features Used	Accuracy
T1, T2 and DTI	$77 \pm 0.013$
T1 and T2	$73 \pm 0.006$

# Structured and DTI features vs Structured features only

T1-T2 features vs Combined T1-T2-DTI features



Features Used	AUC
T1, T2 and DTI	0.756
T1 and T2	0.697

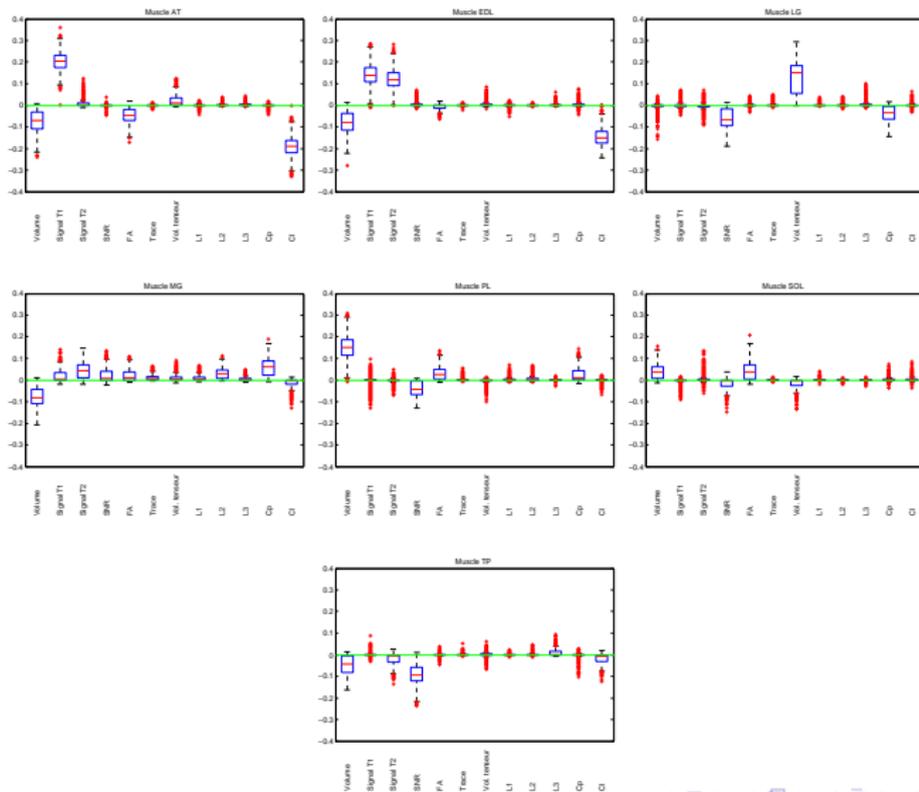
  

Features Used	Accuracy
T1, T2 and DTI	$77 \pm 0.013$
T1 and T2	$73 \pm 0.006$

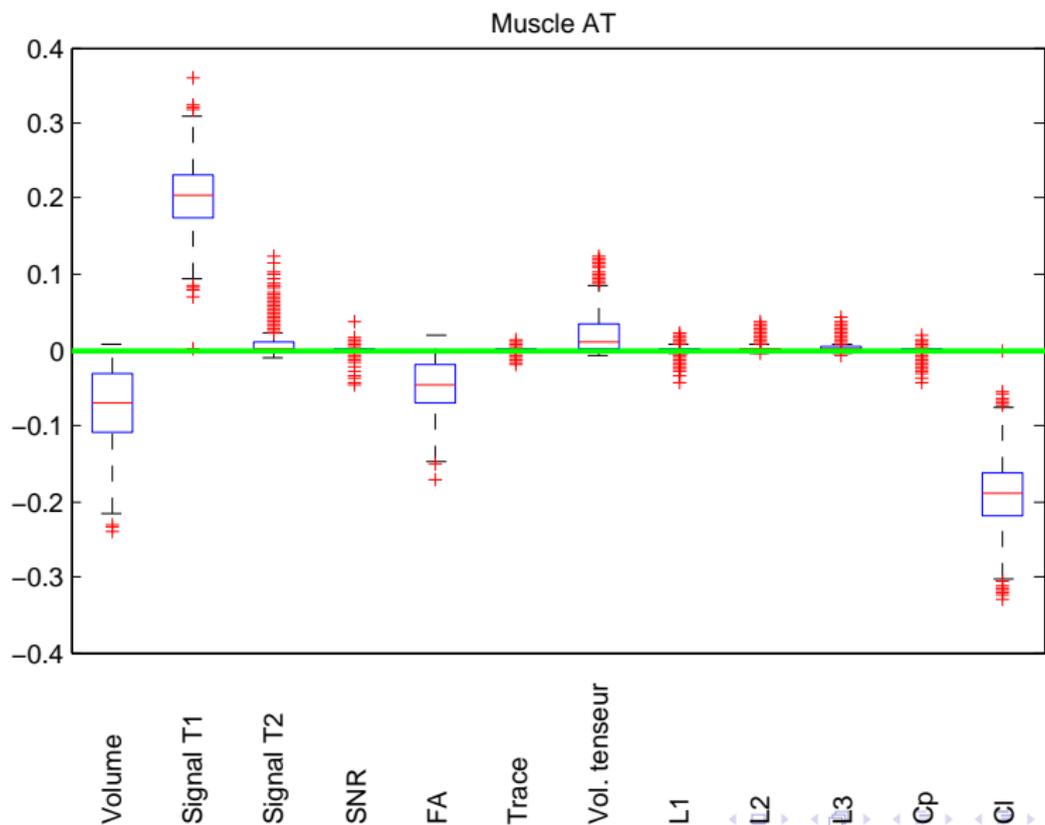
Structured and DTI features vs Structured features only

Wilcoxon signed rank test with p-value  $\ll 0.05$

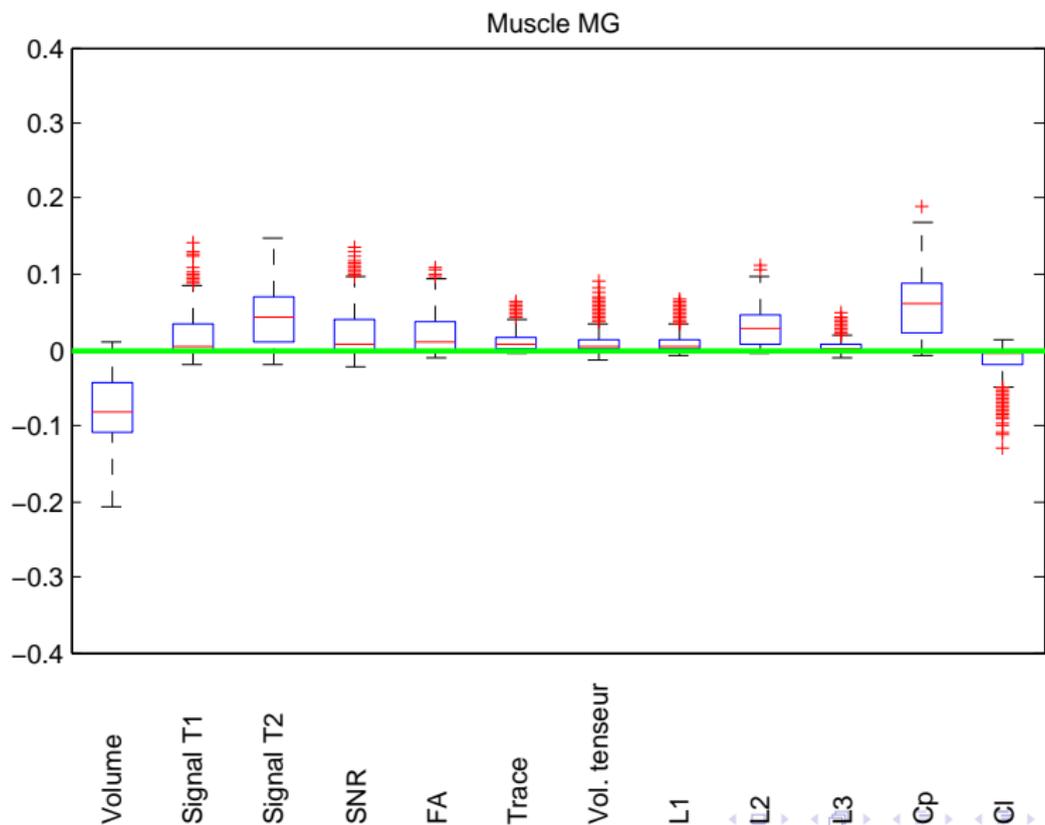
# Feature Evaluation



# Feature Evaluation on anterior tibialis



# Feature Evaluation on medial gastrocnemius



## Conclusion

- We studied the more difficult and clinically relevant task of discriminating between two myopathies (FSH vs DM1).
- MRI markers, and DTI tensor features in particular, can discriminate between disease conditions.
- Sparsity regularization appears to be a more important property of the learning algorithm than non-linearity.
- We introduced a novel machine learning algorithm, the *ksup*-SVM.
- The *ksup*-SVM achieved a mean accuracy of 77%.

Source code is available

<https://gitorious.org/ksup-svm>

# Questions

## Acknowledgements

This work is partially funded by

- the European Research Council under the Seventh Framework Programme (FP7/2007-2013)/ERC Grant 259112.
- the AFM-Telethon foundation.



-  Rina Foygel Andreas Argyriou & Nathan Srebro.  
*Sparse Prediction with the  $k$ -Support Norm.*  
In Advances in Neural Information Processing Systems (NIPS), 2012.
-  A. Argyriou, R. Foygel & N. Srebro.  
*Sparse Prediction with the  $k$ -Support Norm.*  
In NIPS. 2012.
-  Francis R. Bach.  
*Graph kernels between point clouds.*  
In Proceedings of the 25th international conference on Machine learning, International Conference on Machine Learning '08, pages 25–32, 2008.
-  A. Bartels & S. Zeki.  
*The chronoarchitecture of the human brain—natural viewing conditions reveal a time-based anatomy of the brain.*  
NeuroImage, vol. 22, no. 1, pages 419 – 433, 2004.

-  A. Bartels & S. Zeki.  
*Brain dynamics during natural viewing conditions—A new guide for mapping connectivity in vivo.*  
NeuroImage, vol. 24, no. 2, pages 339–349, 2005.
-  M.B. Blaschko, J.A. Shelton & A. Bartels.  
*Augmenting Feature-driven fMRI Analyses: Semi-supervised learning and resting state activity.*  
In NIPS. 2009.
-  M.B. Blaschko, J.A. Shelton, A. Bartels, C.H. Lampert & A. Gretton.  
*Semi-supervised kernel canonical correlation analysis with application to human fMRI.*  
Pattern Recognition Letters, vol. 32, no. 11, pages 1572 – 1583, 2011.

 Karsten M. Borgwardt & Hans-Peter Kriegel.

*Shortest-Path Kernels on Graphs.*

In Proceedings of the Fifth IEEE International Conference on Data Mining, ICDM '05, pages 74–81, Washington, DC, USA, 2005. IEEE Computer Society.

 M.K. Carroll, G.A. Cecchi, I. Rish, R. Garg & A.R. Rao.

*Prediction and interpretation of distributed neural activity with sparse models.*

NeuroImage, vol. 44, no. 1, pages 112 – 122, 2009.

 Fabrizio Costa & Kurt De Grave.

*Fast neighborhood subgraph pairwise distance kernel.*

In Proceedings of the 26th International Conference on Machine Learning, pages 255–262, 2010.

-  Craig J Galban, Stefan Maderwald, Kai Uffmann, Armin de Greiff & Mark E Ladd.  
*Diffusive sensitivity to muscle architecture: a magnetic resonance diffusion tensor imaging study of the human calf.*  
European journal of applied physiology, vol. 93, no. 3, pages 253–262, 2004.
-  Thomas Gärtner, Peter Flach & Stefan Wrobel.  
*On Graph Kernels: Hardness Results and Efficient Alternatives.*  
In Bernhard Schölkopf & Manfred K. Warmuth, editors, Learning Theory and Kernel Machines, volume 2777 of *Lecture Notes in Computer Science*, pages 129–143. Springer Berlin Heidelberg, 2003.
-  Richard J Gilbert & Vitaly J Napadow.  
*Three-dimensional muscular architecture of the human tongue determined in vivo with diffusion tensor magnetic resonance imaging.*  
Dysphagia, vol. 20, no. 1, pages 1–7, 2005.

-  Kristen Grauman & Trevor Darrell.  
*The Pyramid Match Kernel: Efficient Learning with Sets of Features.*  
Journal of Machine Learning Research, vol. 8, pages 725–760, May 2007.
-  D.R. Hardoon, J. Mourão-Miranda, M. Brammer & J. Shawe-Taylor.  
*Unsupervised analysis of fMRI data using kernel canonical correlation.*  
NeuroImage, vol. 37, no. 4, pages 1250 – 1259, 2007.
-  Tamás Horváth, Thomas Gärtner & Stefan Wrobel.  
*Cyclic pattern kernels for predictive graph mining.*  
In Proceedings of the tenth ACM SIGKDD international conference on Knowledge discovery and data mining, KDD '04, pages 158–167, 2004.
-  Madhura Ingahlakar, Drew Parker, Luke Bloy, Timothy PL Roberts & Ragini Verma.  
*Diffusion based abnormality markers of pathology: Toward learned diagnostic prediction of ASD.*  
Neuroimage, vol. 57, no. 3, pages 918–927, 2011.



Stefan Klöppel, Cynthia M Stonnington, Carlton Chu, Bogdan Draganski, Rachael I Scahill, Jonathan D Rohrer, Nick C Fox, Clifford R Jack, John Ashburner & Richard SJ Frackowiak.

*Automatic classification of MR scans in Alzheimer's disease.*

Brain, vol. 131, no. 3, pages 681–689, 2008.



Stephen LaConte, Stephen Strother, Vladimir Cherkassky *et al.*

*Support vector machines for temporal classification of block design fMRI data.*

NeuroImage, vol. 26, no. 2, page 317, 2005.



Zhiqiang Lao, Dinggang Shen, Zhong Xue, Bilge Karacali, Susan M Resnick & Christos Davatzikos.

*Morphological classification of brains via high-dimensional shape transformations and machine learning methods.*

Neuroimage, vol. 21, no. 1, pages 46–57, 2004.

-  P. Mahé, N. Ueda, T. Akutsu, J.-L. Perret & J.-P. Vert.  
*Extensions of marginalized graph kernels.*  
In Proceedings of the Twenty-First International Conference on Machine Learning (ICML 2004), pages 552–559, 2004.
-  Pierre Mahé & Jean-Philippe Vert.  
*Graph kernels based on tree patterns for molecules.*  
Machine Learning, vol. 75, no. 1, pages 3–35, 2009.
-  B. Ng, V. Siless, G. Varoquaux, J.-B. Poline, B. Thirion & R. Abugharbieh.  
*Connectivity-informed Sparse Classifiers for fMRI Brain Decoding.*  
In Pattern Recognition in Neuroimaging, 2012.
-  Liva Ralaivola, Sanjay Joshua Swamidass, Hiroto Saigo & Pierre Baldi.  
*Graph kernels for chemical informatics.*  
Neural Networks, vol. 18, no. 8, pages 1093–1110, 2005.



Jan Ramon & Thomas Gaertner.

*Expressivity versus efficiency of graph kernels.*

In Proceedings of the First International Workshop on Mining Graphs, Trees and Sequences, pages 65–74, 2003.



N. Shervashidze, S. V. N. Vishwanathan, T. Petri, K. Mehlhorn & K. Borgwardt.

*Efficient graphlet kernels for large graph comparison.*

In Proceedings of the International Workshop on Artificial Intelligence and Statistics. Society for Artificial Intelligence and Statistics, 2009.



Nino Shervashidze, Pascal Schweitzer, Erik Jan van Leeuwen, Kurt Mehlhorn & Karsten M. Borgwardt.

*Weisfeiler-Lehman Graph Kernels.*

Journal of Machine Learning Research, vol. 12, pages 2539–2561, November 2011.

-  S. Song, Z. Zhan, Z. Long, J. Zhang & L. Yao.  
*Comparative Study of SVM Methods Combined with Voxel Selection for Object Category Classification on fMRI Data.*  
PLoS One, vol. 6, no. 2, page e17191, 2011.
-  R. Tibshirani.  
*Regression Shrinkage and Selection via the Lasso.*  
Journal of the Royal Statistical Society Series B, vol. 58, pages 267–288, 1996.
-  S. Vichy N. Vishwanathan, Nicol N. Schraudolph, Risi Imre Kondor & Karsten M. Borgwardt.  
*Graph Kernels.*  
Journal of Machine Learning Research, vol. 11, pages 1201–1242, 2010.



Boris Weisfeiler & A.A. Lehman.

*A reduction of a graph to a canonical form and an algebra arising during this reduction.*

Nauchno-Technicheskaya Informatsia, vol. 2, no. 9, pages 12–16, 1968.



H. Zou & T. Hastie.

*Regularization and variable selection via the elastic net.*

Journal of the Royal Statistical Society Series B, vol. 67, no. 2, pages 301–320, 2005.



Hui Zou & Trevor Hastie.

*Regularization and variable selection via the Elastic Net.*

Journal of the Royal Statistical Society, Series B, vol. 67, pages 301–320, 2005.